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AUTHOR Masalski, William J.

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ABSTRACT

This book seeks to develop, enhance, and expand students' understanding of mathematics by using technology. Topics covered include the advantages of spreadsheets along with the opportunity to explore the 'what if?' type of questions encountered in the problem-solving process, enhancing the user's insight into the development and use of algorithms, allowing students to see a progression of calculations, and more. (ASK)

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ED 430 789

How to

>>>> USE THE SPREADSHEET AS A TOOL
>>>>>>> IN THE SECONDARY SCHOOL
>>>>>>>> MATHEMATICS CLASSROOM
>>>>>>>>>>>>>>>>>>>> Second Edition

>(For Windows and Macintosh Operating Systems)

>>>>> William J. Masalski

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HOW TO USE THE SPREADSHEET AS A TOOL IN THE SECONDARY SCHOOL MATHEMATICS CLASSROOM

Second Edition

(FOR WINDOWS AND MACINTOSH OPERATING SYSTEMS)

William J. Masalski
University of Massachusetts Amherst,
Amherst, Massachusetts

National Council of Teachers of Mathematics
Reston, Virginia

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INTRODUCTION

In "The Use of Technology in the Learning and Teaching of Mathematics," a 1994 position statement of the National Council of Teachers of Mathematics (NCTM), the NCTM stated the following:

> Technology is changing the ways in which mathematics is used and is driving the creation of new fields of mathematical study. Consequently, the content of mathematics programs and the methods by which mathematics is taught and learning assessed are changing. The ability of teachers to use the tools of technology to develop, enhance, and expand students' understanding of mathematics is crucial. These tools include computers, appropriate calculators (scientific, graphing, programmable, etc.), videodisks, CD-ROM, telecommunications networks by which to access and share real-time data, and other emerging educational technologies. Exploration of the perspectives these tools provide on a wide variety of topics is required by teachers.
>
> It is the position of the National Council of Teachers of Mathematics that the use of the tools of technology is integral to the learning and teaching of mathematics. Continual improvement is needed in mathematics curricula, instructional and assessment methods, access to hardware and software, and teacher education.

In the *Curriculum and Evaluation Standards for School Mathematics* (1989), the National Council of Teachers of Mathematics has indicated that

> the use of technology in instruction should further alter both the teaching and the learning of mathematics. Computer software can be used effectively for class demonstrations and independently by students to explore additional examples, perform independent investigations, generate and summarize data as part of a project, or complete assignments. Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings. In this setting, the teacher encourages experimentation and provides opportunities for students to summarize ideas and establish connections with previously studied topics.

Some of the advantages of spreadsheets follow:

- The spreadsheet is readily adaptable for problems that are iterative, recursive, or tabular in conceptual format and enables teacher and students to tinker with values of variables, constants, and step size and to explore the tempting "what if?" type of questions encountered in the problem-solving process.
- The spreadsheet can enhance the user's insight into the development and use of algorithms and modeling for the solution of mathematical problems.
- The spreadsheet frees students from being hampered by laborious manipulation of numbers and allows them to concentrate on the mathematical problem itself. This permits the in-depth exploration of meaningful mathematical applications without concern for the possible complex calculations needed to reveal the essence of the mathematics involved.
- The spreadsheet allows the students to see a progression of calculations on the screen as they are generated and permits students to change one variable at a time to see what effect that change has on the overall pattern of those calculations.

How to Use the Spreadsheet as a Tool in the Secondary School Mathematics Classroom—with the accompanying computer disks, one for use with Macintosh computers and one for Windows computers—is an attempt to illustrate by example how the spreadsheet can be used to help realize the goals above.

THE SPREADSHEET: AN INTRODUCTION

The spreadsheet is a mathematical environment accessed through the use of a computer. In simple terms, it is nothing more than a very large rectangular array of boxes or cells, each of which is identified by a unique address. In almost all spreadsheet programs, the address consists of a letter to indicate the column in which the cell is located and a numeral to indicate the row in which the cell is located. For example, the address A3 would be used to indicate that the cell in question is located in column A, row 3 (see fig. 1).

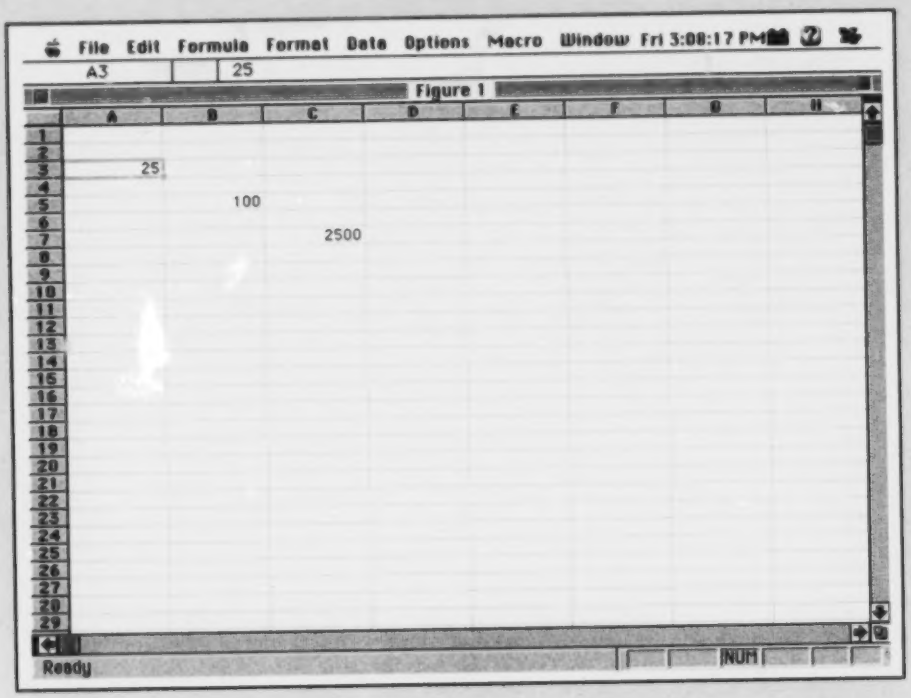

Fig. 1

Note just above the array on the left side of the screen it is indicated that cell A3 is currently the position of the cursor and that the number 25 has been entered directly in that cell.

Figure 1 illustrates but a small portion of an actual spreadsheet application file: some spreadsheets can have hundreds of rows and columns. For example, Microsoft Excel 97 for Windows 95 actually has 256 columns labeled A through IV and 65 536 rows labeled 1 through 65 536. In this text, however, and in most practical classroom applications, very rarely is more than one screen full of rows and columns needed in a single spreadsheet file.

The power of the spreadsheet lies in the fact that each cell can contain a numerical value determined either by simple direct insertion of the value or by a mathematical formula using information obtained from cells anywhere on the spreadsheet file currently under consideration.

Figure 2 illustrates the same spreadsheet file as figure 1 but is obtained by selecting the "show formula" option, which shows *how* the values indicated in the cells were actually calculated: A3 was "calculated" by actually typing in the value 25; B5, as the sum of the number indicated in A3 and the number 75; and C7, as the product of the numbers calculated in A3 and B5. In this "displayed formula mode," the formulas, and the actual typed-in values appear at the left side of each cell.

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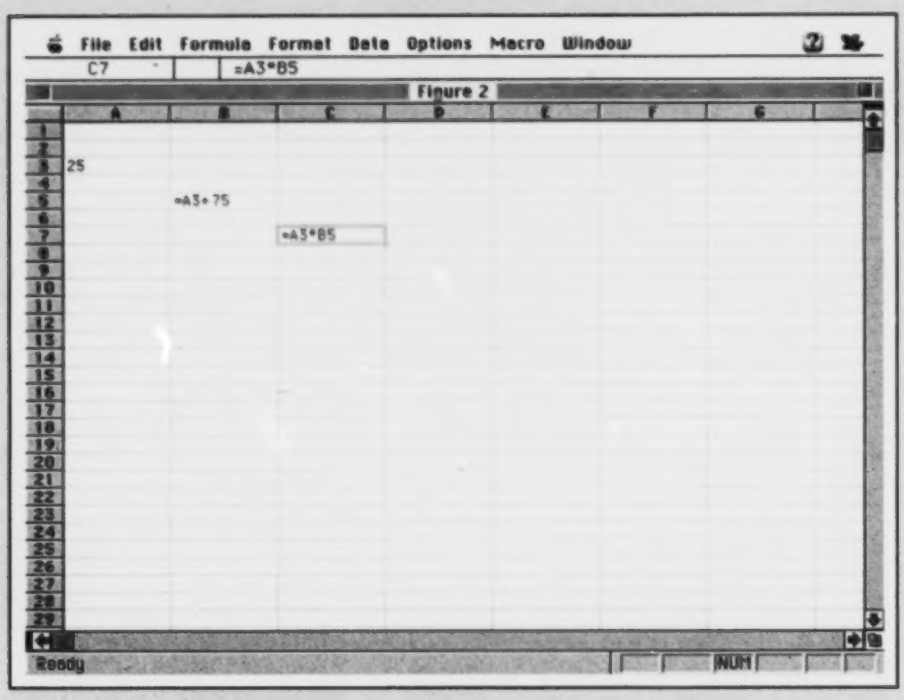

Fig. 2

Note in figure 2 that since the cursor was placed on cell C7 when the screen image was printed, the formula that calculated the value indicated in cell C7 is listed on the left side of the screen just above the array as follows:

=A3*B5

Spreadsheet applications permit the creation of both simple and complex formulas such as those illustrated above as well as many built-in functions such as

=SUM( ),

in which the user indicates inside the parentheses which values are to be summed. The following illustrate options available on most spreadsheet files for completing the SUM function:

=SUM(A3,75), which would sum the number found in cell A3 with the number 75;

=SUM(D2:D10), which would sum the numbers found in the cells in column D from cell D2 to cell D10; and

=SUM(D2,D3,D4,D5,D6,D7,D8,D9,D10), which would also sum the numbers found in the cells in column D from cell D2 to cell D10.

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HOW TO USE THIS BOOK AND SOFTWARE WITH COMPUTERS RUNNING WINDOWS OR MACINTOSH OPERATING SYSTEMS

All the figures in this book are actual "screen dump" images of spreadsheet program files, such as indicated in figure 1. Wheras this book uses images captured from Microsoft Excel, the text is written in general terms and the illustrations would be very similar to, if not exactly the same as, images taken from other spreadsheet applications such as Microsoft Works, ClarisWorks, and others.

All these applications allow the user to switch back and forth between the "displayed calculation mode" and the "displayed formula mode." They also have the capability of "protecting" any cell from accidentally being erased by inadvertently typing in new information.

The two 3 1/2–inch disks that are available with this text contain the spreadsheet files considered in this text. One disk is for Macintosh computers, and the other is for computers running on Windows 95–based or later operating systems. On the following page are suggestions for making modifications of these files. All the files are saved on each disk in SYLK (a universally recognized file format that can be imported by most spreadsheet applications), in Excel 4 or later for the Macintosh, in Excel 97 for Windows, and in Microsoft Works 4 for both Macintosh and Windows.

The spreadsheet files considered in this text are but representative of those that can be developed to explore mathematics using this electronic environment. Each file can be used independently of any of the others, but several are designed to complement others. They are all designed to serve as models for the myriad of similar files that you and your students will develop on your own.

Some of the files and accompanying notes, such as "Magic Arrays" and "Volume," permit the students to explore the tempting "what if?" type of questions encountered in the problem-solving process. Others, such as "Completing the Square" and the "Bisection Algorithm," are intended to demonstrate how the spreadsheet can be used as an electronic chalkboard to present a variety of algorithms clearly for closer examination and exploration by the students. All the files allow the students to change values of variables, constants, and step size easily and offer opportunities to enhance their insight into the development and use of algorithms and modeling for solving mathematical problems.

The spreadsheet files are arranged by mathematical topic. They proceed, in general, from fairly simple activities appropriate for the middle school level to topics appropriate for the precalculus classroom. The Exploration Ideas for Students section of each topic gives answers to the questions raised. The teacher should note the purpose of the activity and peruse the exploration ideas to determine if the activity is appropriate for his or her students. The next step for the teacher is to provide any necessary background or lead-up discussion on the topic before engaging the students with the indicated exploration ideas. The teacher might use the spreadsheet as an electronic chalkboard to introduce the topic and then have students work alone or in pairs at the computer. There is not one correct way to use these materials.

Note: Although this text will provide suggestions for writing, adapting, and using spreadsheets, it is not designed to teach in detail how to operate the various spreadsheet programs. For help in becoming familiar with spreadsheet programs, consult the guides that accompany those programs.

THE DIFFERENCES BETWEEN THE MACINTOSH AND WINDOWS VERSIONS

The screen images and directions for repeating or initiating a new calculation in spreadstheet applications differ slightly between the Macintosh and Windows versions. In general—

on the Macintosh, the key combination is <command> <=>;
on Windows Excel, the key combination is <F9> or <control> <=F9>; and
on Windows Works, the key is just <F9>.

The above are all indicated directly on the spreadsheet files for each version. The SYLK files emulate the Excel versions but may need to be changed when they are imported into other spreadsheet applications.

EXCEL SETTINGS

This text will refer to Excel with the following general settings active:

Excel 4.+: Options menu/Workspace/in Display/deselect R1C1.
Options menu/Calculation/in Calculation/select Automatic.
Options menu/Calculation/in Iteration/check iteration and indicate
1 in Maximum Iterations and .001 in Maximum Change.

Excel 97: Tools menu/Options/General Tab/deselect R1C1.
Tools menu/Options/Calculation Tab/Calculation/select Automatic.
Tools menu/Options/Calculation Tab/check iteration and indicate
1 in Maximum Iterations and .001 in Maximum Change.

CONVERTING FROM SYLK FORMAT FILES

When converting from SYLK to ClarisWorks, you may need to change the following:

Under the Page Setup menu, change from Portrait to Landscape view.

Under the Options menu, select Display and deselect Mark Circular References.

Instead of using the <command> <=> keys as indicated on the screen to repeat an action, you need to use the <shift> <command> <=> keys.

Similar adjustments may be needed for other spreadsheet applications.

CONVERTING BETWEEN "DISPLAYED CALCULATIONS" AND "DISPLAYED FORMULA" MODES

Excel 4.+: Options menu/Display/in Cells, select or deselect Formulas.

Excel 97: Tools menu/Options/Views Tab/in Window options/select or deselect Formulas.

Microsoft Works: View menu/select or deselect Formulas.

GRAPHICS

Only the Excel files on the accompanying disks contain the graphics illustrated with some of the topics in this text. The method of creating "charts," as they are usually called, differs rather significantly from application to application, so the user must determine how these graphics were created and how the application in use might reproduce them.

TOPIC: Magic Arrays

SPREADSHEET FILE: Magic Arrays Excel (Macintosh and Windows)
Magic Arrays Works (Macintosh and Windows)
Magic Arrays SYLK (Macintosh and Windows)

PURPOSE: To explore the nature of magic arrays, investigate how they work, use an environment that promotes the development of problem-solving skills, and—for those who need it—provide practice in addition, subtraction, multiplication, and division

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays a three-by-three array with each cell containing a zero and the sum of each row and column being equal to zero (fig. 3). As the value in any given cell is changed, the sums of the cells in both the row and the column containing that cell automatically change to reflect the new value.

CELL FORMULAS:

H6 =SUM(B6:F6)
H10 =SUM(B10:F10)
H14 =SUM(B14:F14)
B18 =SUM(B6:B14) D18 =SUM(D6:D14) F18 =SUM(F6:F14)

EXPLORATION IDEAS FOR STUDENTS:

1. Using the digits 1 through 9 once and only once in the nine boxes, try to complete the magic array so that the sum of each of the rows and each of the columns is the same. How many ways can you do this?
2. Using any sequence of nine numbers once and only once in the nine boxes, try to complete the magic square so that the sum of each of the rows and each of the columns is the same. How many ways can you do this?
3. After finding a magic array, what happens if you add the same number to each of the cells? What happens when you complete this task?
4. After finding a magic array, what happens if you subtract, multiply, or divide (select just one operation at a time) each cell by the same number? What happens when you complete this task?
5. Write a formula in cell H2 that will calculate the sum of the three numbers in that diagonal of the array. Do the same for cell H18 and that diagonal. Now try completing this "really" magic array so that all the rows, columns, and diagonals have the same sum.
6. Repeat items 1–4 above after completing item 5, this time so that all the rows, columns, *and diagonals* have the same sum.
7. Try writing a four-by-four magic array; a five-by-five magic array. What happens if you subtract, multiply, or divide (select just one operation at a time) each cell in these magic arrays by the same number?

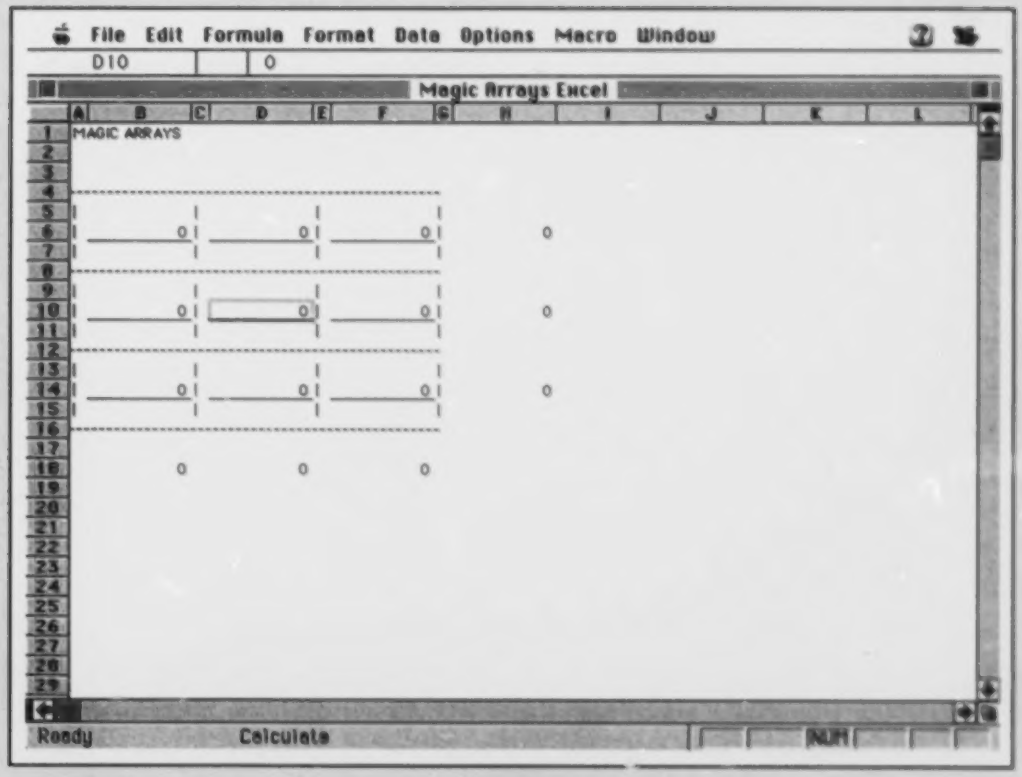

Fig. 3

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TOPIC: Collecting and Analyzing Data A

SPREADSHEET FILE: Collect & Analyze Data A Excel (Macintosh and Windows)
Collect & Analyze Data A Works (Macintosh and Windows)
Collect & Analyze Data A SYLK (Macintosh and Windows)

PURPOSE: To explore the collection and analysis of data by making use of tables and graphs

CONTENT OR GRADE LEVEL: Middle school and prealgebra

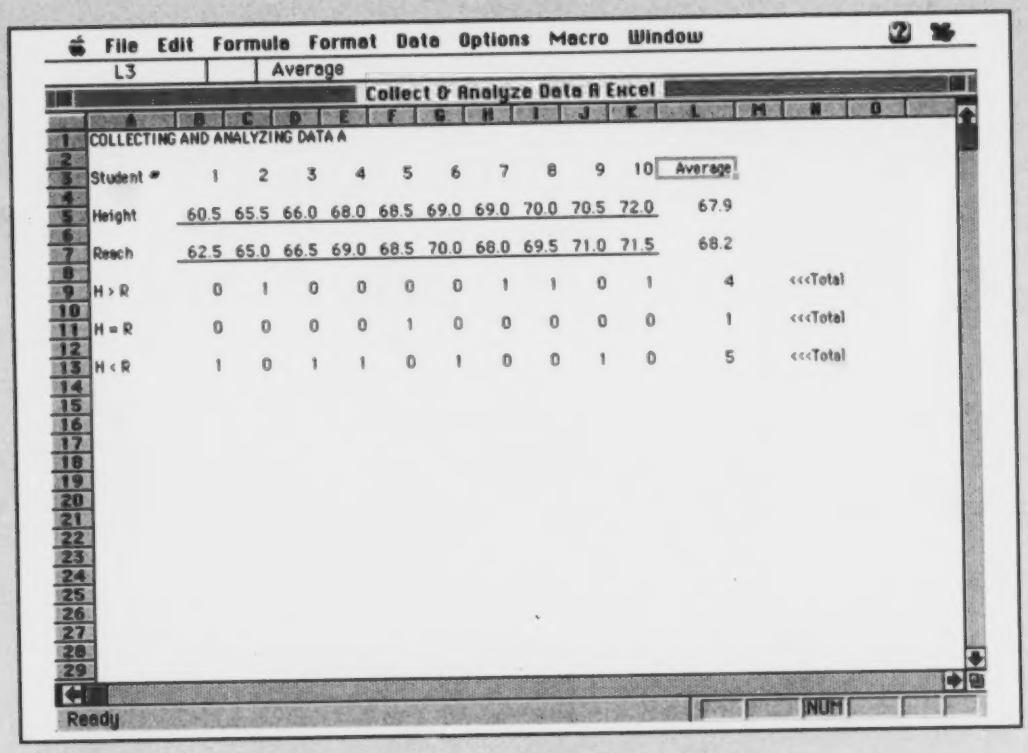
DESCRIPTION: This file displays a table for inserting the height and reach of ten students. It then calculates the average height and reach and determines in turn the number of students whose height is less than, equal to, and greater than their reach. The information is then displayed using a scatter graph (fig. 4).

CELL FORMULAS:

L5	=AVERAGE(B5:K5)				
L7	=AVERAGE(B7:K7)				
B9	=IF(B5>B7,1,0)	...	K9	=IF(K5>K7,1,0)	L9 =SUM(B9:K9)
B11	=IF(B5=B7,1,0)	...	K11	=IF(K5=K7,1,0)	L11 =SUM(B11:K11)
B13	=IF(B5<B7,1,0)	...	K13	=IF(K5<K7,1,0)	L13 =SUM(B13:K13)

EXPLORATION IDEAS FOR STUDENTS:

1. Measure the height without shoes of ten students in your class.
2. Record their heights from shortest to tallest in the spreadsheet table.
3. Measure the reach from fingertip to fingertip of the same ten students. To be sure they are each reaching as far as possible, take the measurement as they put their backs against a wall and hold their arms out parallel to the floor and touching the wall.
4. Record their reaches in the spreadsheet table.
5. Which student has the shortest reach? The longest reach? Which students have a longer reach than height? A shorter reach than height? The same reach as height? What is the average reach and height of these ten students?
6. Repeat the activity for ten other students and compare your results.
7. What observation can you make about the average height and reach of students in your class?
8. Repeat the activity with a class of students in a younger grade level; an older grade level. Compare the results.


File Edit Formula Format Data Options Macro Window
L3
Average
Collect & Analyze Data A Excel
COLLECTING AND ANALYZING DATA A
Student #
1 2 3 4 5 6 7 8 9 10 Average
Height
60.5 65.5 66.0 68.0 68.5 69.0 69.0 70.0 70.5 72.0 67.9
Reach
62.5 65.0 66.5 69.0 68.5 70.0 68.0 69.5 71.0 71.5 68.2
H > R
0 1 0 0 0 0 1 1 0 1 4 <<<Total
H = R
0 0 0 0 1 0 0 0 0 0 1 <<<Total
H < R
1 0 1 1 0 1 0 0 1 0 5 <<<Total
Ready
NUM

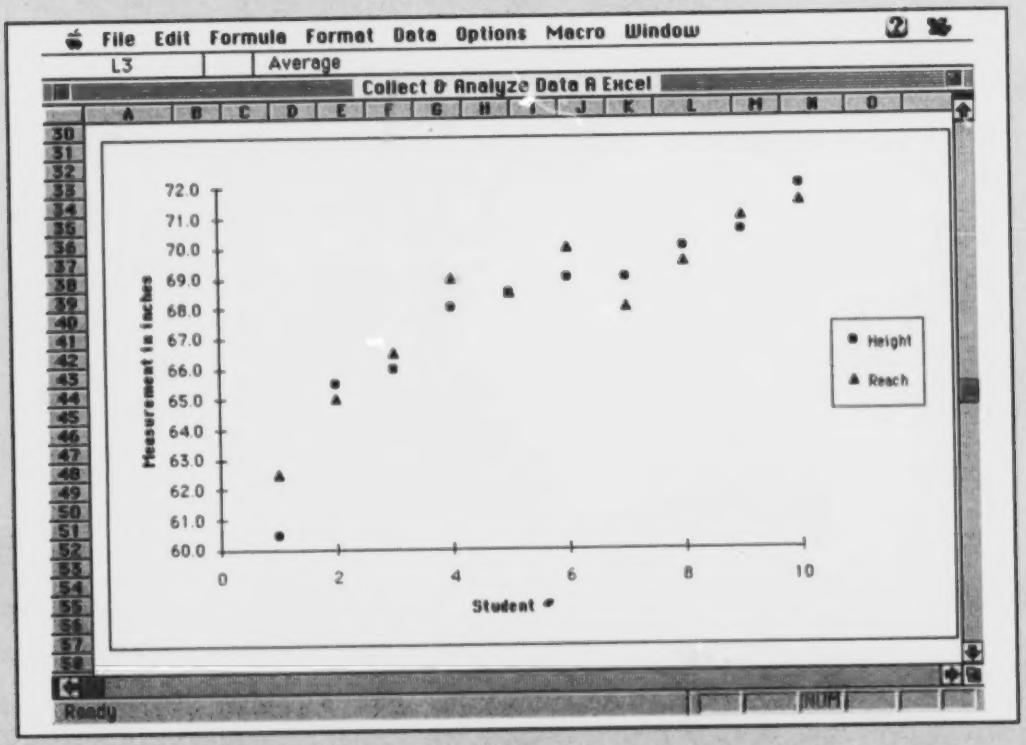


Fig. 4

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TOPIC: Collecting and Analyzing Data B

SPREADSHEET FILE: Collect & Analyze Data B Excel (Macintosh and Windows)
Collect & Analyze Data B Works (Windows only)
Collect & Analyze Data B SYLK (Macintosh and Windows)

PURPOSE: To explore the collection and analysis of data by making use of tables and graphs

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays a table for inserting responses to five questions, each asked of ten students. It then calculates the average response for each question in terms of all respondents, male respondents, and female respondents. The information is then displayed using a scatter-line graph (fig. 5).

CELL FORMULAS:

C15 =AVERAGE(C4:C13) ... G15 =AVERAGE(G4:G13)

C16 =SUM(IF(B4=1,C4),IF(B5=1,C5),IF(B6=1,C6),IF(B7=1,C7),IF(B8=1,C8),
IF(B9=1,C9),IF(B10=1,C10),IF(B11=1,C11),IF(B12=1,C12),
IF(B13=1,C13))
/(COUNT(IF(B4=1,1,A33),IF(B5=1,1,A33),IF(B6=1,1,A33),IF(B7=1,1,A33),
IF(B8=1,1,A33),IF(B9=1,1,A33), IF(B10=1,1,A33), IF(B11=1,1,A33),
IF(B12=1,1,A33), IF(B13=1,1,A33))

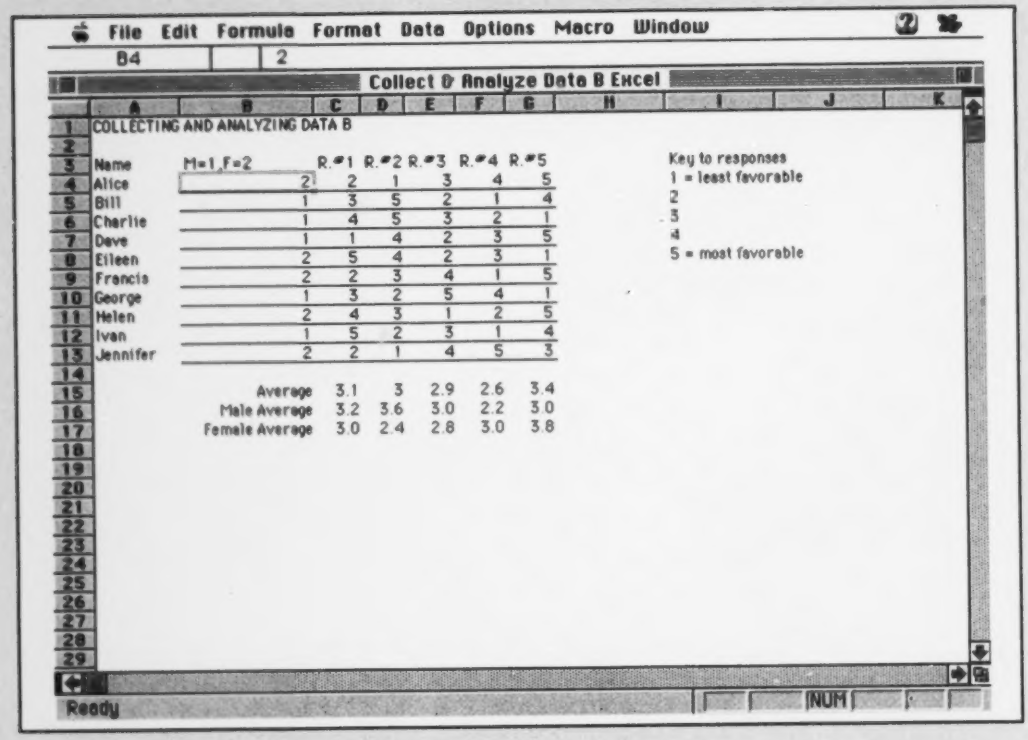
D16 through G16 Replace each of the C's above by a D, E, F, or G respectively.

C17 =SUM(IF(B4=2,C4),IF(B5=2,C5),IF(B6=2,C6),IF(B7=2,C7),IF(B8=2,C8),
IF(B9=2,C9),IF(B10=2,C10),IF(B11=2,C11),IF(B12=2,C12),
IF(B13=2,C13))
/(COUNT(IF(B4=2,1,A33),IF(B5=2,1,A33),IF(B6=2,1,A33),IF(B7=2,1,A33),
IF(B8=2,1,A33),IF(B9=2,1,A33), IF(B10=2,1,A33), IF(B11=2,1,A33),
IF(B12=2,1,A33), IF(B13=2,1,A33))

D17 through G17 Replace each of the C's above by a D, E, F, or G respectively.

EXPLORATION IDEAS FOR STUDENTS:

1. Develop a survey form on which you list five different soft drinks. Ask ten students to rank the drinks from 1 to 5 with 1 being their least favorite and 5 their most favorite.
2. Enter the information on the spreadsheet table, also indicating in the appropriate column whether the respondent is a male (1) or female (2).
3. What is the most popular drink among everyone surveyed? Among the males? Among the females?
4. Which drink got the highest average rating? The lowest average rating? How did male and female averages compare to the average of all respondents?
5. What other questions can you investigate from this survey?
6. Repeat the same survey with ten other students. Compare your results.
7. Repeat the activity with a different set of five survey items.


File Edit Formula Format Data Options Macro Window
B4
2
Collect & Analyze Data B Excel
COLLECTING AND ANALYZING DATA B
Name
M=1,F=2
R.#1 R.#2 R.#3 R.#4 R.#5
Alice 2 2 1 3 4 5
Bill 1 3 5 2 1 4
Charlie 1 4 5 3 2 1
Dave 1 1 4 2 3 5
Eileen 2 5 4 2 3 1
Francis 2 2 3 4 1 5
George 1 3 2 5 4 1
Helen 2 4 3 1 2 5
Ivan 1 5 2 3 1 4
Jennifer 2 2 1 4 5 3
Average 3.1 3 2.9 2.6 3.4
Male Average 3.2 3.6 3.0 2.2 3.0
Female Average 3.0 2.4 2.8 3.0 3.8
Key to responses
1 = least favorable
2
3
4
5 = most favorable
Ready
NUM

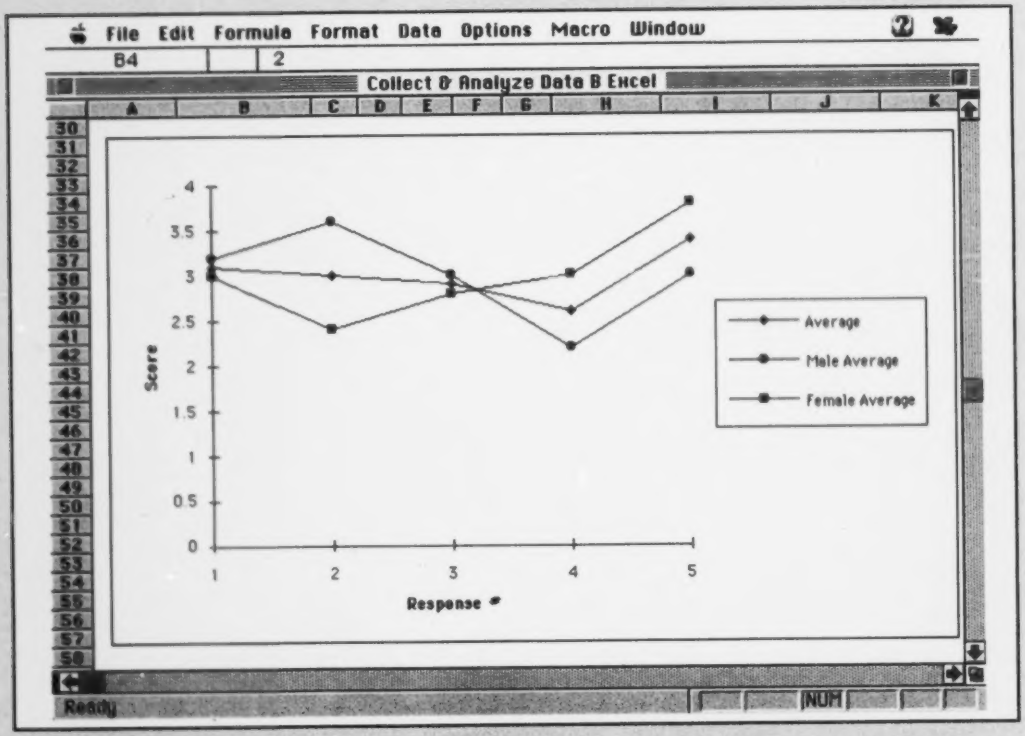


Fig. 5

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TOPIC: Collecting and Analyzing Data C

SPREADSHEET FILE: Collect & Analyze Data C Excel (Macintosh and Windows)
Collect & Analyze Data C Works (Macintosh and Windows)
Collect & Analyze Data C SYLK (Macintosh and Windows)

PURPOSE: To explore the collection and analysis of data, using tables and graphs

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays a table for inserting responses to ten trials of random sampling from a bag containing a given number (25 in the example that follows) of red, yellow, blue, and green tiles (fig. 6). The actual number of tiles of each color is not known to the participants. After selecting ten tiles, indicating on the table the number of tiles of each color drawn, and returning the drawn tiles to the bag, the students guess the number of tiles of each color that are actually in the bag. This process is continued for ten trials.

CELL FORMULAS:

F6	=SUM(B6:E6)	...	F15	=SUM(B15:E15)			
L6	=SUM(H6:K6)	...	L15	=SUM(H15:K15)			
B17	=SUM(B6:B15)	...	E17	=SUM(E6:E15)		F17	=SUM(B17:E17)
H17	=SUM(H6:H15)	...	K17	=SUM(K6:K15)		L17	=SUM(H17:K17)
B19	=AVERAGE(B6:B15)	...	E19	=AVERAGE(E6:E15)			
H19	=AVERAGE(H6:H15)	...	K19	=AVERAGE(K6:K15)			

EXPLORATION IDEAS FOR STUDENTS:

1. Mix all the color tiles together in a pile on your desk.
2. With your eyes closed, select 25 tiles and place them in a paper bag.
3. Take ten tiles from the bag without looking in the bag as you do.
4. Count the number of each color and record it on the spreadsheet file.
5. Return the tiles to the bag.
6. Record your guess of the actual number of tiles in the bag.
7. Repeat steps 2–6 above for a total of ten trials.
8. What is your final estimate of the number of tiles of each color in the bag?
9. Count the number of tiles of each color in the bag, and compare your final estimate with the actual number in the bag.
10. Repeat this activity using 50 and 100 tiles in the bag.

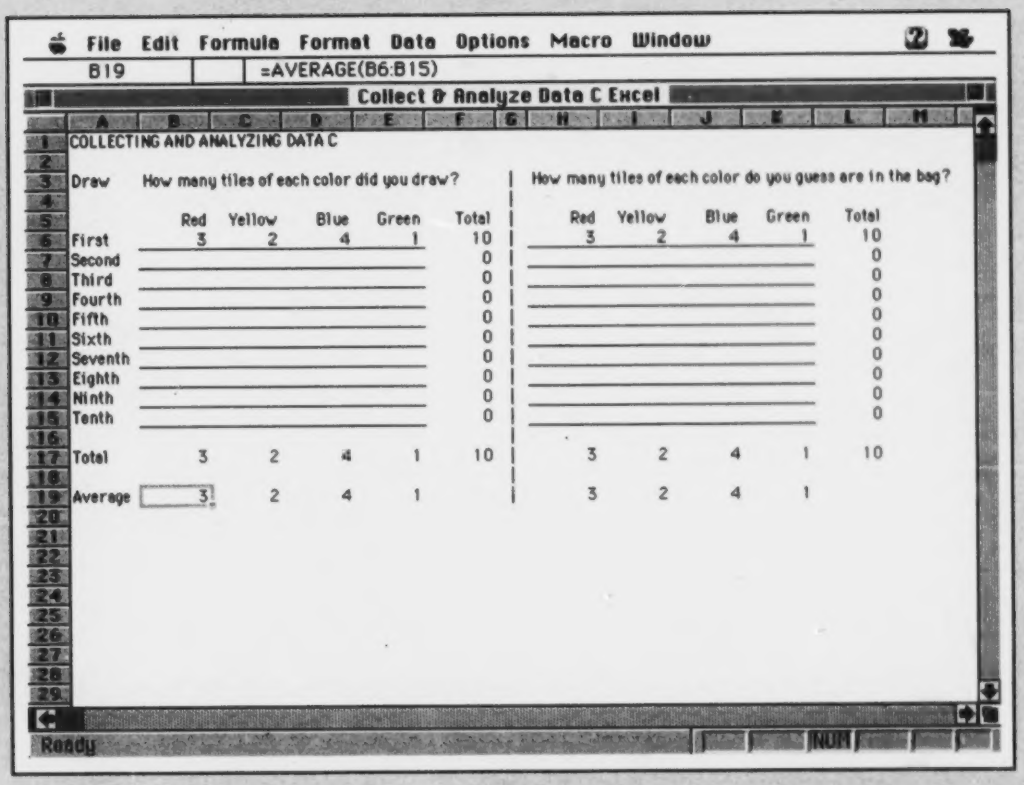

Fig. 6

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TOPIC: Square Roots

SPREADSHEET FILE: Square Roots Excel (Macintosh and Windows)
Square Roots Works (Macintosh and Windows)
Square Roots SYLK (Macintosh and Windows)

PURPOSE: To explore the divide-and-average method for finding square roots of numbers

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays the formula

S = x^2

where *S* represents the square of a given number *x* (fig. 7).

Cell B5 contains a selected value for *S*: 144.0000.

Cell B7 contains a first estimate for *x*: 5.0000.

Cell B11 contains that first estimate of the square root of *S*; cell D11, the quotient of *S* by that first estimate; and cell F11, the average of these two numbers.

CELL FORMULAS:

The $ signs below indicate that when a formula was copied from the cell above, that part containing the dollar signs was copied in absolute terms. The default copying of a cell reference is in relative terms.

B11 =B7	D11 =B5/B7	F11 =0.5*(B11+D11)
B12 =F11	D12 =$B$5/B12	F12 =0.5*(B12+D12)
B13 =F12	D13 =$B$5/B13	F13 =0.5*(B13+D13)
B14 =F13	D14 =$B$5/B14	F14 =0.5*(B14+D14)
B15 =F14	D15 =$B$5/B15	F15 =0.5*(B15+D15)
...	...	...

EXPLORATION IDEAS FOR STUDENTS:

1. How does this spreadsheet work? Place the cursor in each of the cells displaying numbers to reveal how they were generated.
2. What happens to the quotient if your first estimate for the square root is too large? Too small?
3. How many divisions were necessary to find the square root of 144 when our estimate was 5?
4. Change the value of *S* in cell B7 to 383. How many divisions are necessary to find the square root of 383 when our estimate for *x* is 5?
5. Use the up and down arrows to scroll up and down column A and see further approximations for the square root of the number under consideration.
6. Use this file to find the square root of 12 345 678. What would be a good first estimate for *x*?
7. What is the minimum number of divisions that must be made to determine the actual square root of a number? Under what conditions will that happen?
8. How can we minimize the number of divisions necessary to come very close to, or to determine the actual square root of, any number?

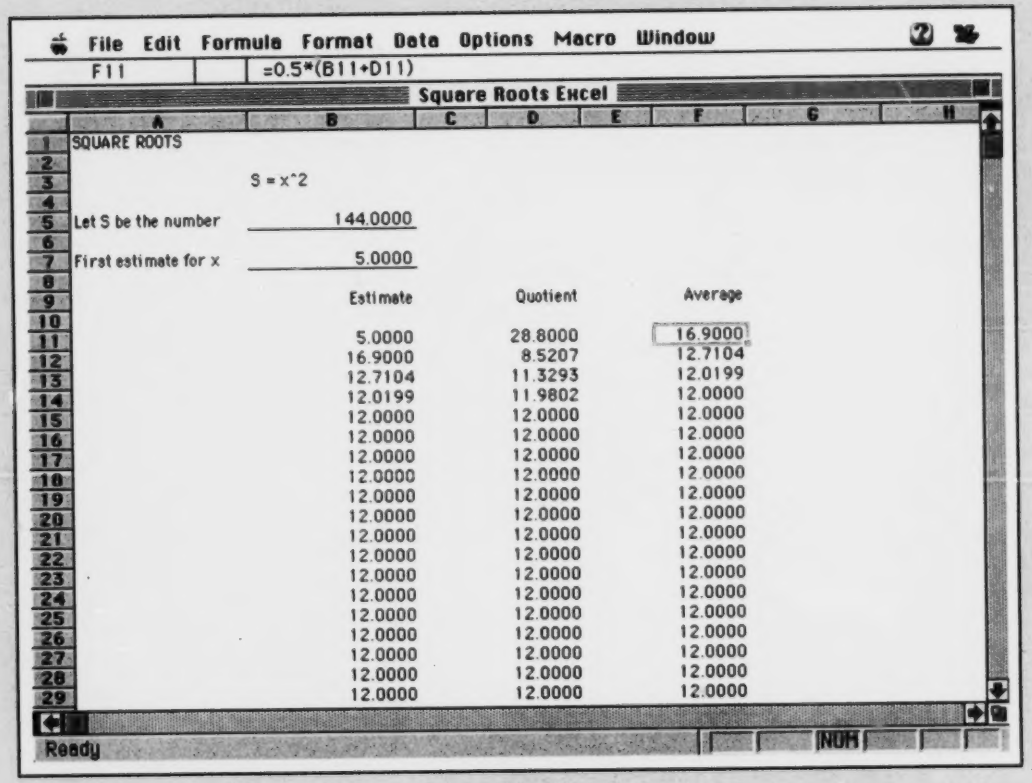

Fig. 7

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TOPIC: Grade Book

SPREADSHEET FILE: Grade Book Excel (Macintosh and Windows)
Grade Book Works (Macintosh and Windows)
Grade Book SYLK (Macintosh and Windows)

PURPOSE: To explore the concept of determining averages and the spreadsheet functions that determine sum, count, minimum, maximum, and average

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays a sample grade book that not only records the quiz grades of each student but also calculates each student's average grade for the quizzes actually taken. For each quiz given, this file indicates the total points for the class, the actual number of students taking that quiz, the average quiz score, and the maximum and minimum quiz scores. (See fig. 8.)

CELL FORMULAS:

G5 =AVERAGE(B5:F5)
G6 =AVERAGE(B6:F6)
G7 =AVERAGE(B7:F7)
G8 =AVERAGE(B8:F8)
G9 =AVERAGE(B9:F9)

C14 =SUM(C4:C13)	D14 =SUM(D4:D13)	E14 =SUM(E4:E13)
C15 =COUNT(C4:C13)	D15 =COUNT(D4:D13)	E15 =COUNT(E4:E13)
C16 =C14/C15	D16 =D14/D15	E16 =E14/E15
C17 =MIN(C5:C9)	D17 =MIN(D5:D9)	E17 =MIN(E5:E9)
C18 =MAX(C5:C9)	D18 =MAX(D5:D9)	E18 =MAX(E5:E9)

EXPLORATION IDEAS FOR STUDENTS:

1. Who received the highest score for Quiz 2? How many students actually took Quiz 3? What was the average score for Quiz 1?

For questions 2, 3, and 4 that follow, first try to determine the answers without using this electronic grade book, then use it to check your answer.

2. What if Allen earned a 92 on his make-up for Quiz 1? What would be his average for the three quizzes?
3. How many points would Carol have to earn on her make-up for Quiz 3 in order to raise her average to 70 for the three quizzes?
4. What if Carol earned exactly the number of points needed to raise her average to 70? What would be the new class average for Quiz 3?
5. Add a new student, Frances, to the class and give her scores of 70, 83, and 69 for the first three quizzes. What is her average?
6. Study each of the formulas in the rows 14, 15, 16, 17, and 18 and in column G. Try to determine how they work.
7. Use this spreadsheet file as a model to create another one that will allow you to keep track of your own quiz and homework grades.

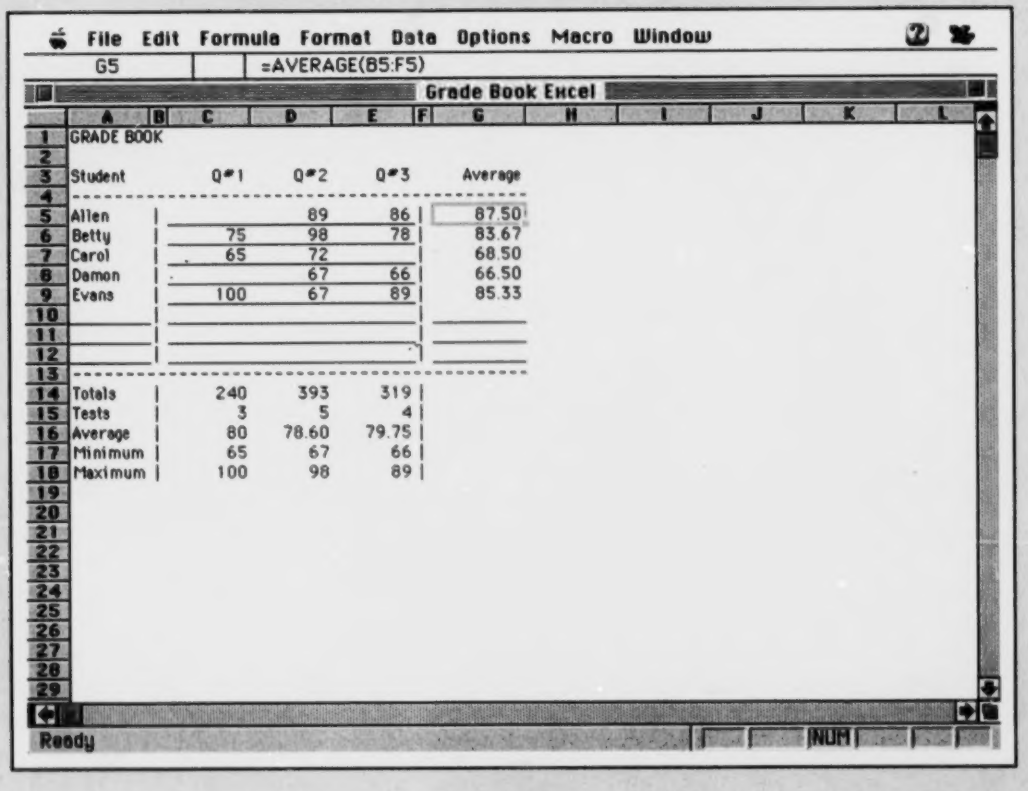

Fig. 8

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TOPIC: Multiples and Prime Numbers A

SPREADSHEET FILE: Multiples A Excel (Macintosh and Windows)
Multiples A Works (Macintosh and Windows)
Multiples A SYLK (Macintosh and Windows)

PURPOSE: To investigate the multiples of any number entered and to develop a listing of the prime numbers in the manner of the sieve of Eratosthenes

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays a ten-column-by-fourteen-row array of the counting numbers from 2 to 140—except that in the example illustrated (fig. 9), the multiples of 11 greater than 11 have been replaced by 0s. It also indicates that the user is to enter a 1 to reset the file or a number to try—in this example the number 11 has been entered. As developed, the file will replace each multiple of the number entered with a 0.

The columns are not wide enough to display all of each formula. As has been done in the screen dump, however, by placing the cursor on any cell (C5, for example), we can read at the bottom of the screen the formula for any cell in those columns.

Consider the formulas in the array, using the formula in C5 as an example:

=IF(M3=1,2,=IF(=AND(M3<>C5,C5/M3=INT(C5/M3)),0,C5))

If M3=1, the value 2 will be placed in cell C5. Otherwise, the following inner conditional is considered:

=IF(=AND(M3<>C5,C5/M3=INT(C5/M3)),0,C5)

In the inner conditional above, if *both* the relations M3<>C5 *and* C5/M3=INT(C5/M3) are true, a 0 will be placed in cell C5. Otherwise what was in C5 will remain there. In essence the formula checks first to see if the file is being reset by the insertion of a 1 in M3. If so, cell C5 will be reset to its default value of 2. If any other value is placed in M3, the inner conditional checks first to see if that value is 2, the same as the value in C5. If it is a 2, the relation is false and the 2 is retained in cell C5. If it is not a 2 *and* it is evenly divisible by the value in M3, it must be a multiple of 2, and a 0 is placed in the cell.

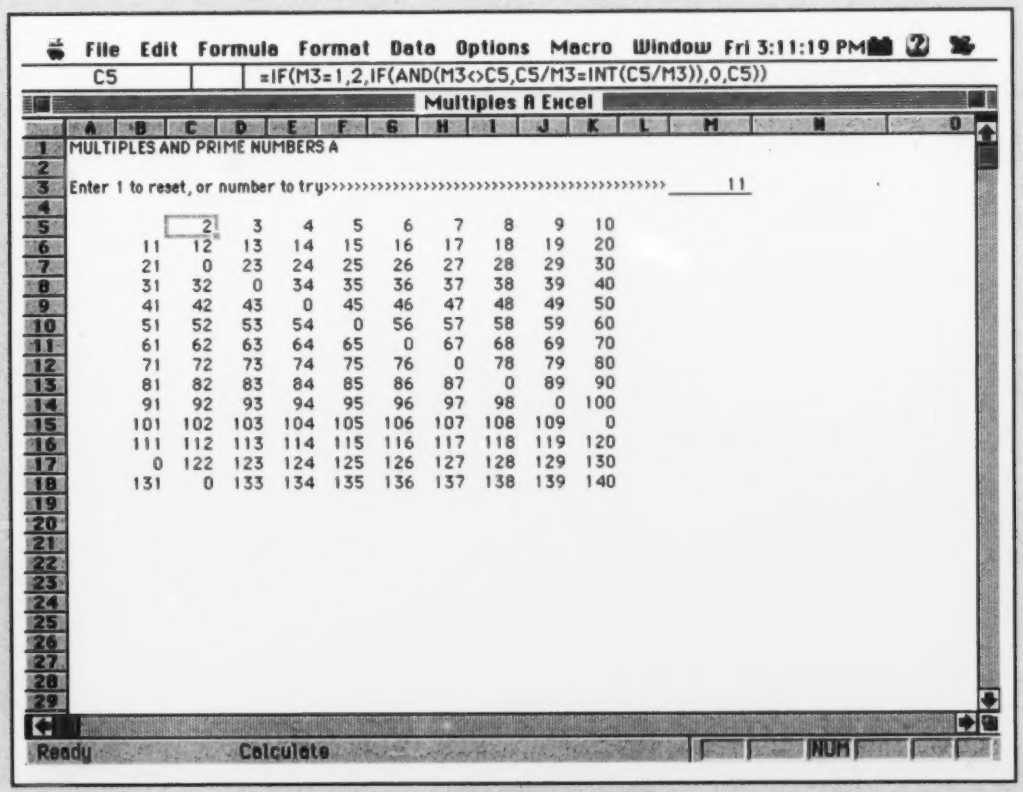
CELL FORMULAS:

C5 =IF(M3=1,2,=IF(=AND(M3<>C5,C5/M3=INT(C5/M3)),0,C5))
D5 =IF(M3=1,3,=IF(=AND(M3<>C5,C5/M3=INT(C5/M3)),0,D5))
E5 =IF(M3=1,4,=IF(=AND(M3<>C5,C5/M3=INT(C5/M3)),0,E5))
...

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file by entering a 1 in cell M3. Enter the number 3 and press <return>. What happens? How are all the cells that have been blanked out by zeros related?
2. Reset the file, enter the number 2, and press <return>. How are the blanked-out cells related?
3. Reset the file, and then enter in succession the counting numbers greater than 1 until no more cells become blanked out.
4. Which numbers entered do not cause numbers in the array to be blanked out?
5. What was the last number you entered that was able to cause the blanking out of a cell that did not yet have a 0 in it?

6. What is the largest counting number you need to try to ensure that the file has blanked out all non-primes?
7. How are the resulting cells related?
8. This file is modeled after the sieve of Eratosthenes. Your textbook may have additional information on the sieve. If not, investigate other sources to read more about it.


File Edit Formula Format Data Options Macro Window Fri 3:11:19 PM
C5
=IF(M3=1,2,IF(AND(M3<>C5,C5/M3=INT(C5/M3)),0,C5))
Multiples A Excel
MULTIPLES AND PRIME NUMBERS A
Enter 1 to reset, or number to try>>>

TOPIC: Multiples and Prime Numbers B

SPREADSHEET FILE: Multiples B Excel (Macintosh and Windows)
Multiples B Works (Macintosh and Windows)
Multiples B SYLK (Macintosh and Windows)

PURPOSE: To investigate the multiples of any number entered or develop a listing of the prime numbers in the manner of the sieve of Eratosthenes; especially useful in studying the concepts of twin primes

CONTENT OR GRADE LEVEL: Middle school and prealgebra

DESCRIPTION: This file displays a six-column-by-fourteen-row array of the counting numbers from 2 to 84—except that in the example illustrated (fig. 10), the multiples of 7 greater than 7 have been replaced by 0s. It also indicates that the user is to enter a 1 to reset the file or a number to try—in this example the number 7 has been entered. As developed, the file will replace each multiple of the number entered with a 0.

The columns are not wide enough to display all of each formula. As has been done in the screen dump, however, by placing the cursor on any cell (C5, for example), we can read at the bottom of the screen the formula for any cell in those columns.

Consider the formulas in the array, using the formula in C5 as an example:

=IF(K3=1,2,=IF(=AND(K3<>C5,C5/K3=INT(C5/K3)),0,C5))

If K3=1, the value 2 will be placed in cell C5. Otherwise the following inner conditional is considered:

=IF(=AND(K3<>C5,C5/K3=INT(C5/K3)),0,C5)

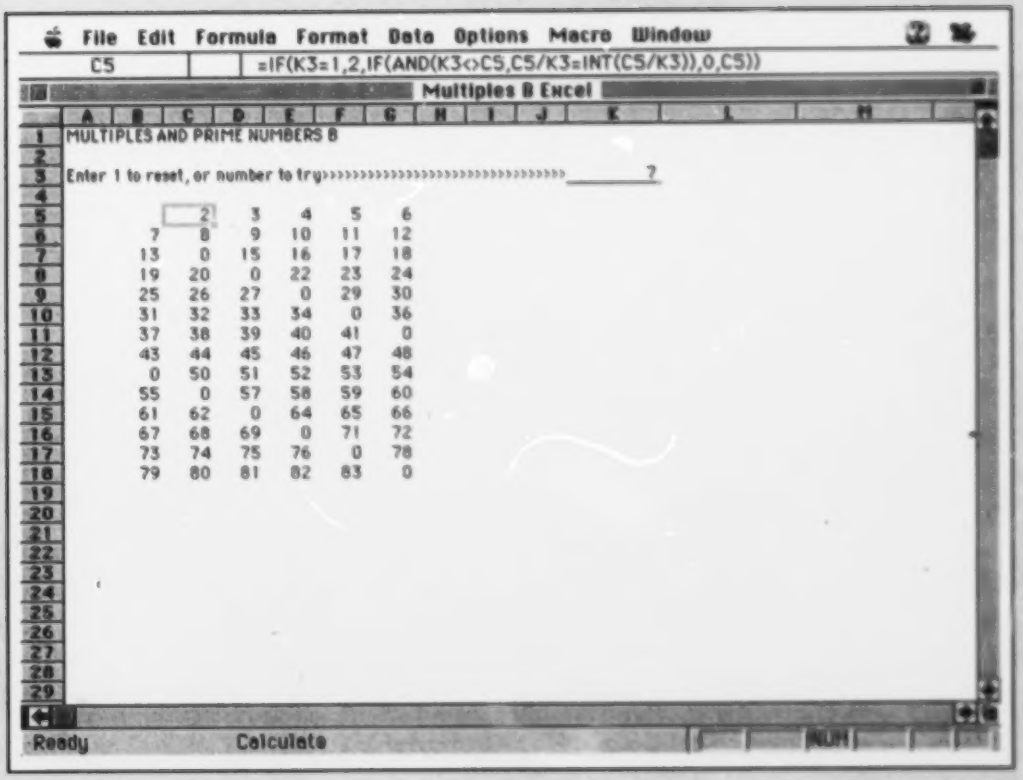
In the inner conditional above, if *both* the relations K3<>C5 *and* C5/K3=INT(C5/K3) are true, a 0 will be placed in cell C5. Otherwise, what was in C5 will remain there. In essence the formula checks first to see if the file is being reset by the insertion of a 1 in K3. If so, cell C5 will reset to its default value of 2. If any other value is placed in K3, the inner conditional checks first to see if that value is the same as the value in C5—2. If it is a 2, the relation is false and the 2 is retained in cell C5. If it is not a 2 *and* it is evenly divisible by the value in K3, it must be a multiple of 2, and a 0 is placed in the cell.

CELL FORMULAS:

C5 =IF(K3=1,2,=IF(=AND(K3<>C5,C5/K3=INT(C5/K3)),0,C5))
D5 =IF(K3=1,3,=IF(=AND(K3<>C5,C5/K3=INT(C5/K3)),0,D5))
E5 =IF(K3=1,4,=IF(=AND(K3<>C5,C5/K3=INT(C5/K3)),0,E5))
...

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file by entering a 1 in cell K3. Enter the number 2 and press <return>. How are all the cells that have been blanked out by zeros related?
2. Without resetting the file, enter the number 3 and press <return>. How are the newly blanked-out cells related?
3. After blanking out all the multiples of 2 and 3, what generalization can be made about the numbers still left unblanked?
4. Twin primes are pairs of primes separated by only one counting number, such as 5 and 7. How does your response to the previous question relate to the concept of twin primes?
5. Continue to enter in succession the counting numbers greater than 3 until no more cells are blanked out. Study this list and identify all the pairs of twin primes listed.


File Edit Formula Format Data Options Macro Window
C5
=IF(K3=1,2,IF(AND(K3<>C5,C5/K3=INT(C5/K3)),0,C5))
Multiples B Excel
MULTIPLES AND PRIME NUMBERS B
Enter 1 to reset, or number to try>>>

TOPIC: Compound Interest A

SPREADSHEET FILE: Interest A Excel (Macintosh and Windows)
Interest A Works (Macintosh and Windows)
Interest A SYLK (Macintosh and Windows)

PURPOSE: To investigate the determination of compound interest and its effect on annual yield

CONTENT OR GRADE LEVEL: High school algebra through precalculus

DESCRIPTION: This file states in row 3 that only the original principal, rate, and compounding periods per year in row 7 can be changed (fig. 11). As any one of the indicated numbers is changed, this file automatically calculates the new balance in the account at the end of each of the first 30 years of the investment. Although only the first 11 years are shown on the screen when the file is loaded into the computer, the other rows indicating years 12 through 30 can be scrolled onto the screen as needed by the up and down arrow keys. To calculate these values, the file uses the formula as indicated in E7:

=A7*((1+(B7/C7))^(C7*D7))

This translates to

$$\text{New balance} = P\left(1+\frac{r}{n}\right)^{nt}$$

where P = principal, r = annual rate, n = periods per year compounded, and t = number of years.

CELL FORMULAS:

D7	1	E7	=A7*((1+(B7/C7))^(C7*D7))
D9	=D7+1	E9	=A7*((1+(B7/C7))^(C7*D9))
D10	=D9+1	E10	=A7*((1+(B7/C7))^(C7*D10))
D11	=D10+1	E11	=A7*((1+(B7/C7))^(C7*D11))
...		...	

EXPLORATION IDEAS FOR STUDENTS:

1. At interest generated only once a year (compounded annually), how long will it take for an investment of $100 to double in value if invested at an annual rate of 6 percent? What if you invest $38.50 under these conditions?
2. If interest at 6 percent is generated twice a year (compounded semiannually), how long will it take for an investment of $100 to double in value?
3. How many times a year would you have to compound the interest at 6 percent to have the investment double in value in eleven years?
4. Suppose you wanted to double your investment of $100 in six years. At what interest rate would you have to invest the money if the interest was generated annually?
5. The effective annual yield is the actual increase in value of an investment in one year. What is the effective annual yield of $1 invested at 8 percent, compounded quarterly?
6. Use this spreadsheet to explore other questions like the above.

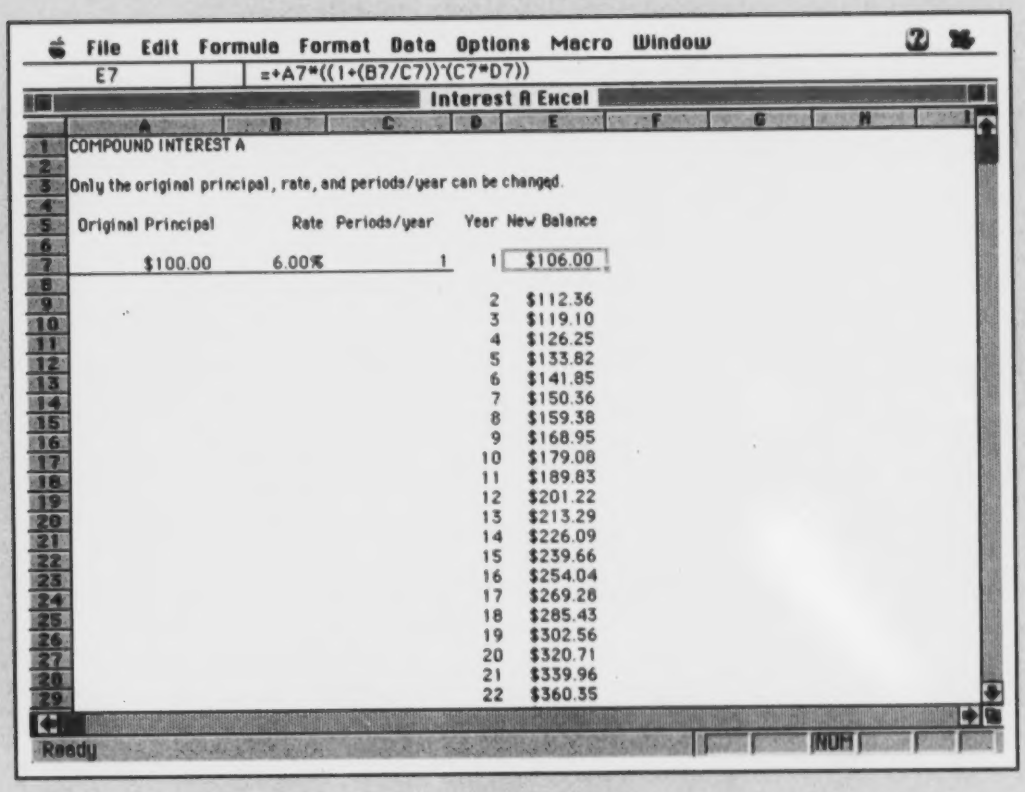


Fig. 11

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TOPIC: Compound Interest B

SPREADSHEET FILE: Interest B Excel (Macintosh and Windows)
Interest B Works (Macintosh and Windows)
Interest B SYLK (Macintosh and Windows)

PURPOSE: To investigate the determination of compound interest and its effect on annual yield

CONTENT OR GRADE LEVEL: High school algebra through precalculus

DESCRIPTION: Whereas the file Interest A considers the balance for a given investment for each year over a 30-year period, this file indicates how changing the number of periods in a year that the interest is compounded affects the balance at the end of the investment years (fig. 12).

This file states in row 3 that only the original principal, rate, and number of years in row 7 can be changed. As any one of these numbers is changed, this file automatically calculates the new balance in the account at the end of the selected number of years that the money is invested. For example, to calculate these values for semiannual compounding of interest, the file makes use of the formula as indicated in E8:

=A7*((1+(B7/2)^(2*D7))

This translates to

$$\text{New balance} = P\left(1+\frac{r}{2}\right)^{2t}$$

where P = principal, r = annual rate, 2 (in this formula) = periods per year, and t = number of years.

CELL FORMULAS:

E7 =A7*((1+(B7/1))^(1*D7))
E8 =A7*((1+(B7/2))^(2*D7))
E9 =A7*((1+(B7/4))^(4*D7))
E10 =A7*((1+(B7/12))^(12*D7))
E11 =A7*((1+(B7/52))^(52*D7))
E12 =A7*((1+(B7/365))^(365*D7))
E13 =A7*((1+(B7/8760))^(8760*D7))
E14 =A7*((1+(B7/525600))^(525600*D7))
E15 =A7*((1+(B7/31536000))^(31536000*D7))
E16 =A7*((1+(B7/315360000))^(315360000*D7))
E17 =A7*((1+(B7/3153600000))^(3153600000*D7))

EXPLORATION IDEAS FOR STUDENTS:

1. The effective annual yield is the actual increase in value of an investment in one year. What is the effective annual yield of $1 invested at 6 percent compounded yearly? Monthly? Daily? Every .01 second?
2. How does the number of times a year that the interest is compounded change the effective annual yield?
3. How much money would have to be invested at 10 percent interest compounded daily in order to realize a gain of at least $100 more than investing the same amount of money for the same time at 10 percent compounded annually?
4. Money invested at 10 percent compounded semiannually is multiplied each year by $(1.05)^2$. What is it multiplied by if the compounding is done quarterly?
5. Use this spreadsheet to consider other questions like the above.

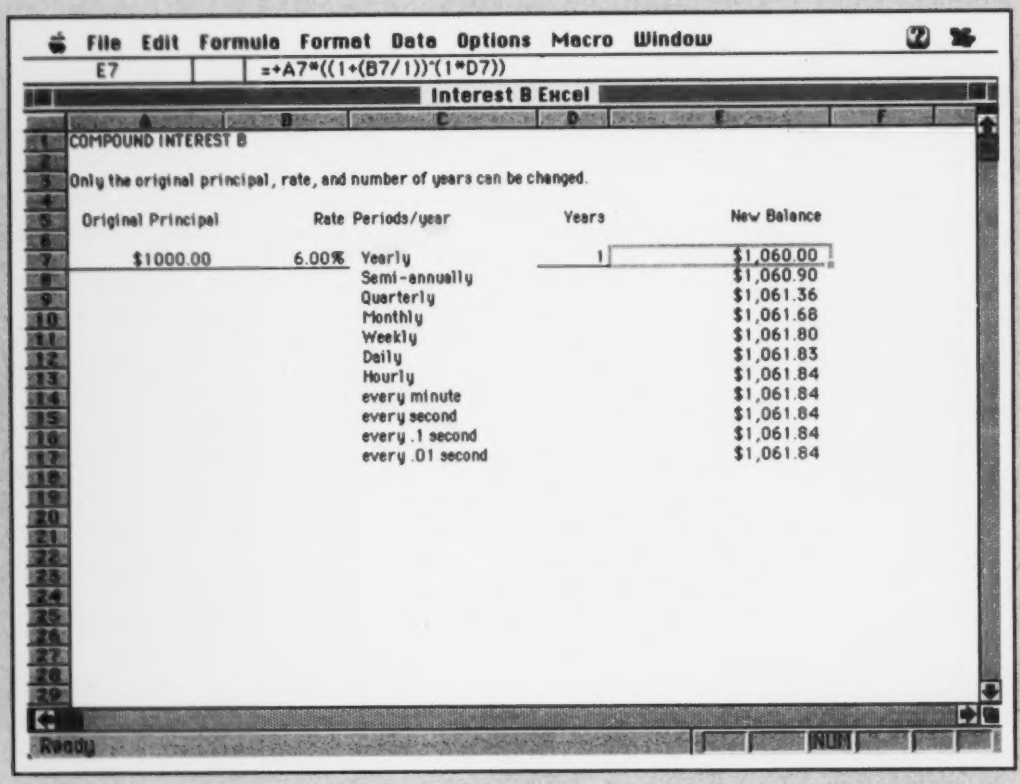

Fig. 12

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TOPIC: Linear Equations with One Equation and One Unknown

SPREADSHEET FILE: Linear A Excel (Macintosh and Windows)
Linear A Works (Macintosh and Windows)
Linear A SYLK (Macintosh and Windows)

PURPOSE: To explore an algorithm for solving a linear equation with one unknown

CONTENT OR GRADE LEVEL: Algebra

DESCRIPTION: This file displays one linear equation with one unknown,

$$2x + 4 = 10,$$

and proceeds, through a series of steps, to simplify and solve the equation (fig. 13). As indicated, only the numbers in row 5 can be changed to create new linear equations with one unknown. A graph is constructed using the information in the block of cells A31:B71 to illustrate the line determined by the equation.

CELL FORMULAS:

A7	=A5	C7	=C5	E7	=C5	G7	=G5	I7	=C5
A9	=A5					G9	=G7-C5		
A11	=A5	C11	=A5			G11	=G9	I11	=A5
						G13	=G9/A5		

A31	-10	B31	=$A$5*A31+$C$5-$G$5
A32	=A31+.5	B32	=$A$5*A32+$C$5-$G$5
...		...	
A71	=A70+.5	B71	=$A$5*A71+$C$5-$G$5

EXPLORATION IDEAS FOR STUDENTS:

For each of lines 1 through 4 below, refer to the file as it is set up with the equation

$$2x + 4 = 10.$$

1. What is done to create the equation in row 7 from the equation in row 5?
2. What is done to create the equation in row 9 from the equation in row 7?
3. What is done to create the equation in row 11 from the equation in row 9?
4. What is done to create the equation in row 13 from the equation in row 11?
5. What happens if you change the numbers in row 5 of the file? Do it and then repeat questions 1 through 4 above.
6. Restate your answers for questions 1 through 4 in general terms.
7. Your correct response to question 6 should result in a method (algorithm) that can be used with paper and pencil—as well as by computer—to solve linear equations of this type.
8. Will this algorithm always work? If so, why? If not, why not?
9. Create other pairs of linear equations and examine their solutions.

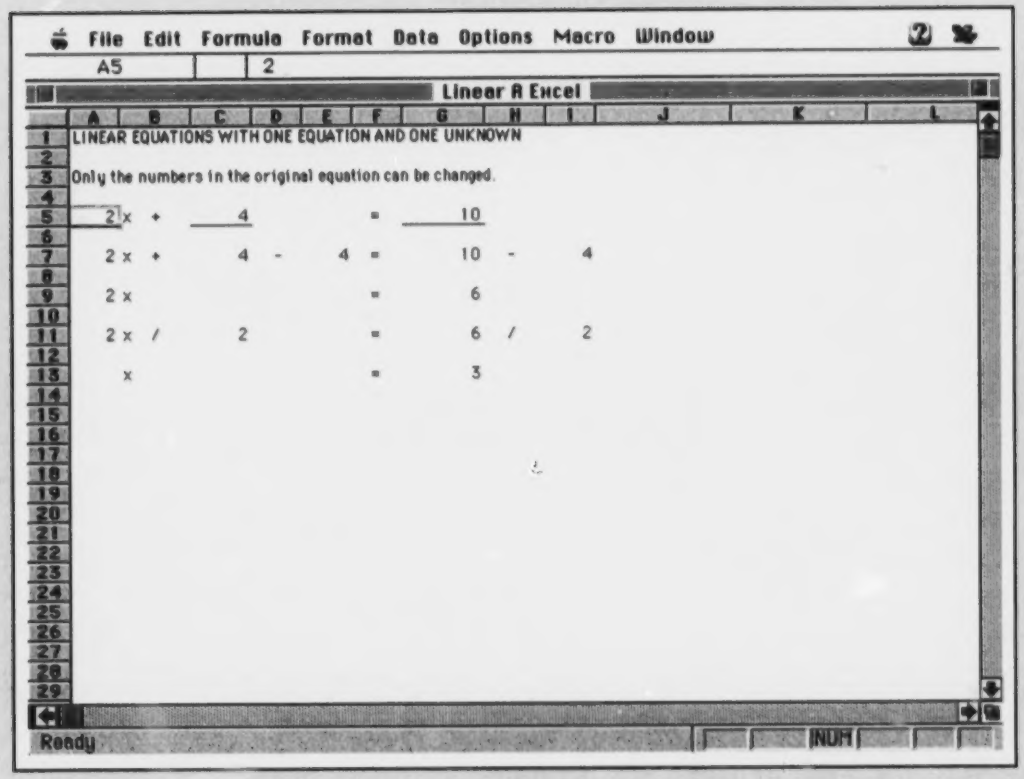
File Edit Formula Format Data Options Macro Window
A5
2
Linear A Excel
LINEAR EQUATIONS WITH ONE EQUATION AND ONE UNKNOWN
Only the numbers in the original equation can be changed.
2x + 4 = 10
2x + 4 - 4 = 10 - 4
2x = 6
2x / 2 = 6 / 2
x = 3
Ready
NUM

Fig. 13

BEST COPY AVAILABLE

TOPIC: Linear Equations with Two Equations and Two Unknowns

SPREADSHEET FILE: Linear B Excel (Macintosh and Windows)
Linear B Works (Macintosh and Windows)
Linear B SYLK (Macintosh and Windows)

PURPOSE: To explore an algorithm for solving a system of two linear equations with two unknowns

CONTENT OR GRADE LEVEL: Algebra through precalculus

DESCRIPTION: This file displays two linear equations with two unknowns,

$2x + 3y = 5$ and $3x + 4y = 6$,

and proceeds, through a series of steps, to find the solution to this system of equations. New systems of two linear equations with two unknowns can be created by entering new coefficients and constants for the equations in lines 5 and 6. A graph is constructed using the information in the block of cells A31:C71to illustrate the intersection of the lines determined by the two equations. (See fig. 14.)

CELL FORMULAS:

A8 =A5/A5	C8 =C5/A5	E8 =E5/A5
A9 =A6	C9 =C6	E9 =E6
A11 =A8	C11 =C8	E11 =E8
A12 =A9-(A9•A8)	C12 =C9-(A9•C8)	E12 =E9-(A9•E8)
A14 =A11	C14 =C11	E14 =E11
A15 =A12	C15 =C12/C12	E15 =E12/C12
A17 =A14-(C14•A15)	C17 =C14-(C14•C15)	E17 =E14-(C14•E15)
A18 =A15	C18 =C15	E18 =E15
A31 -10	B31 =($A$5•A31-$E$5)/-$C$5	C31 =($A$6•A31-$E$6)/-$C$6
A32 =A31+0.5	B32 =($A$5•A32-$E$5)/-$C$5	C32 =($A$6•A32-$E$6)/-$C$6
...	...	...
A71 =A70+0.5	B71 =($A$5•A71-$E$5)/-$C$5	C71 =($A$6•A71-$E$6)/-$C$6

EXPLORATION IDEAS FOR STUDENTS:

1. What is done to create the equation in row 8 from the equation in row 5?
2. What is done to create the equation in row 12 from the equations in rows 8 and 9?
3. What is done to create the equation in row 15 from the equation in row 12? (Although the 0 appears in row 12, do not consider it part of that equation, since it could be removed without effect.)
4. What is done to create the equation in row 17 from the equations in rows 14 and 15? (Although the 0 appears in row 15, do not consider it part of that equation, since it could be removed without effect.)
5. What happens if you enter different coefficients and constants in rows 5 and 6? Make the changes and repeat questions 1 through 4 above.
6. Restate your answers for questions 1 through 4 in general terms.
7. Your correct response to question 6 should result in a method (algorithm) that can be used with paper and pencil—as well as by computer—to solve systems of linear equations.

8. Will this algorithm always work? If so, why? If not, why not?
9. Create other pairs of linear equations and examine their solutions.

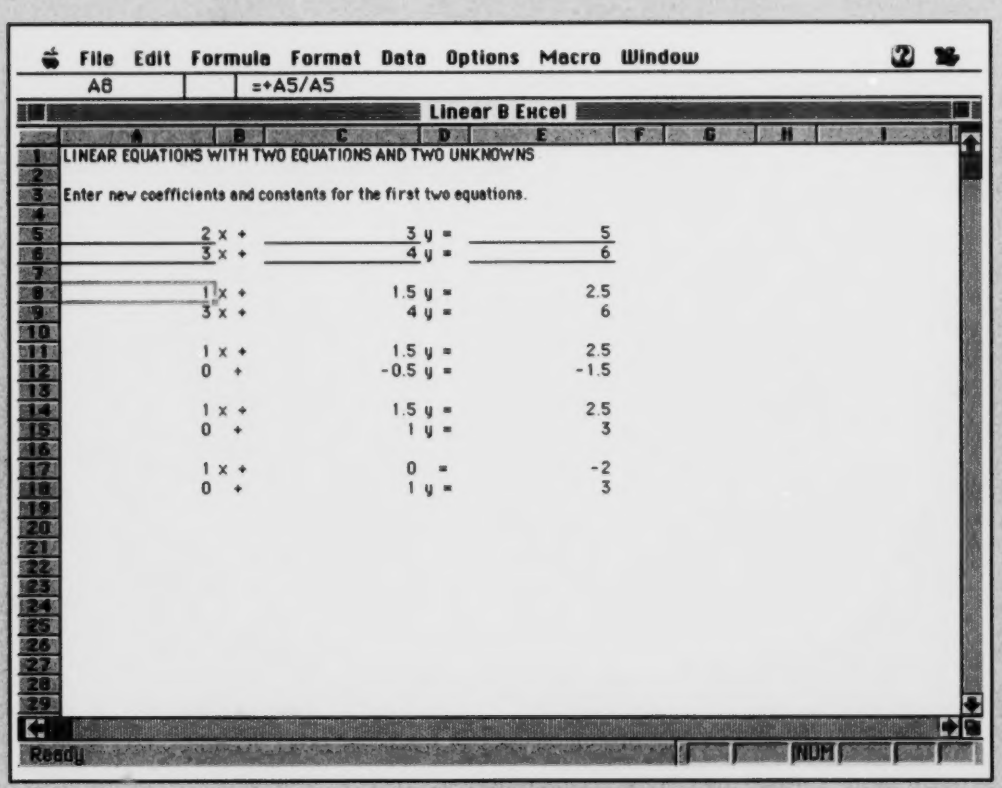
File Edit Formula Format Data Options Macro Window
A8
=+A5/A5
Linear B Excel
LINEAR EQUATIONS WITH TWO EQUATIONS AND TWO UNKNOWNS
Enter new coefficients and constants for the first two equations.
2 x + 3 y = 5
3 x + 4 y = 6
1 x + 1.5 y = 2.5
3 x + 4 y = 6
1 x + 1.5 y = 2.5
0 + -0.5 y = -1.5
1 x + 1.5 y = 2.5
0 + 1 y = 3
1 x + 0 = -2
0 + 1 y = 3
Ready
NUM

Fig. 14

BEST COPY AVAILABLE

TOPIC: Linear Equations with Three Equations and Three Unknowns

SPREADSHEET FILE: Linear C Excel (Macintosh and Windows)
Linear C Works (Macintosh and Windows)
Linear C SYLK (Macintosh and Windows)

PURPOSE: To explore an algorithm for solving a system of three linear equations with three unknowns

CONTENT OR GRADE LEVEL: Algebra through precalculus

DESCRIPTION: This file displays three linear equations with three unknowns,

$$2x + 3y + 4z = 5$$
$$9x + 8y + 7z = 6$$
$$4x + 5y + 4z = -2,$$

and proceeds, through a series of steps, to find the solution to this system. New systems of three linear equations with three unknowns can be created by entering new coefficients and constants for the equations in lines 5, 6, and 7. (See fig. 15.)

CELL FORMULAS:

A9 =A5/A5	C9 =C5/A5	E9 =E5/A5	G9 =G5/A5
A10 =A6	C10 =C6	E10 =E6	G10 =G6
A11 =A7	C11 =C7	E11 =E7	G11 =G7
A13 =A9	C13 =C9	E13 =E9	G13 =G9
A14 =A10-(A10*A9)	C14 =C10-(A10*C9)	E14 =E10-(A10*E9)	G14 =G10-(A10*G9)
A15 =A11-(A11*A9)	C15 =C11-(A11*C9)	E15 =E11-(A11*E9)	G15 =G11-(A11*G9)
A17 =A13	C17 =C13	E17 =E13	G17 =G13
A18 =A14	C18 =C14/C14	E18 =E14/C14	G18 =G14/C14
A19 =A15	C19 =C15	E19 =E15	G19 =G15
A21 =A17-(C17*A18)	C21 =C17-(C17*C18)	E21 =E17-(C17*E18)	G21 =G17-(C17*G18)
A22 =A18	C22 =C18	E22 =E18	G22 =G18
A23 =A19	C23 =C19-(C19*C18)	E23 =E19-(C19*E18)	G23 =G19-(C19*G18)
A25 =A21	C25 =C21	E25 =E21	G25 =G21
A26 =A22	C26 =C22	E26 =E22	G26 =G22
A27 =A23	C27 =C23	E27 =E23/E23	G27 =G23/E23
A29 =A25-(E25*A27)	C29 =C25	E29 =E25-(E25*E27)	G29 =G25-(E25*G27)
A30 =A26	C30 =C26-(E26*C27)	E30 =E26-(E26*E27)	G30 =G26-(E26*G27)
A31 =A27	C31 =C27	E31 =E27	G31 =G27

EXPLORATION IDEAS FOR STUDENTS:

1. Since this spreadsheet file takes more than one full screen to display, a split screen of the file is displayed on page 31. The rest of the file can be observed by scrolling vertically in either of the two screens.

   Note: The screen can be easily split vertically, horizontally, or both by using the mouse key to press and drag the split bar from the top of the vertical slide bar or the left of the horizontal slide bar. By sliding the

vertical bar to row 8, this procedure places the original three equations at the top of the screen. By scrolling the bottom vertical slide bar to expose rows 28 through 31, the solution on the bottom of the screen is indicated:

$2x + 3y + 4z = 5$
$9x + 8y + 7z = 6$
$4x + 5y + 4z = -2$

==========

$1x + 0 + 0 = 2.5$ $(x = 2.5)$
$0 + 1y + 0 = -6$ $(y = -6)$
$0 + 0 + 1z = 4.5$ $(z = 4.5)$

2. By examining the illustration and its discussion as a guide, you will see that the development of the present file parallels that of the previous one, Linear Equations: Two Equations, Two Unknowns.
3. What happens if you change one or more of the coefficients or constants in any one or more of the three equations? Use this spreadsheet file to explore this method for solving three linear equations with three unknowns.
4. Write out the algorithm used. Will it always work? If so, why? If not, why not?
5. Use this file to create and solve other systems of three linear equations with three unknowns.

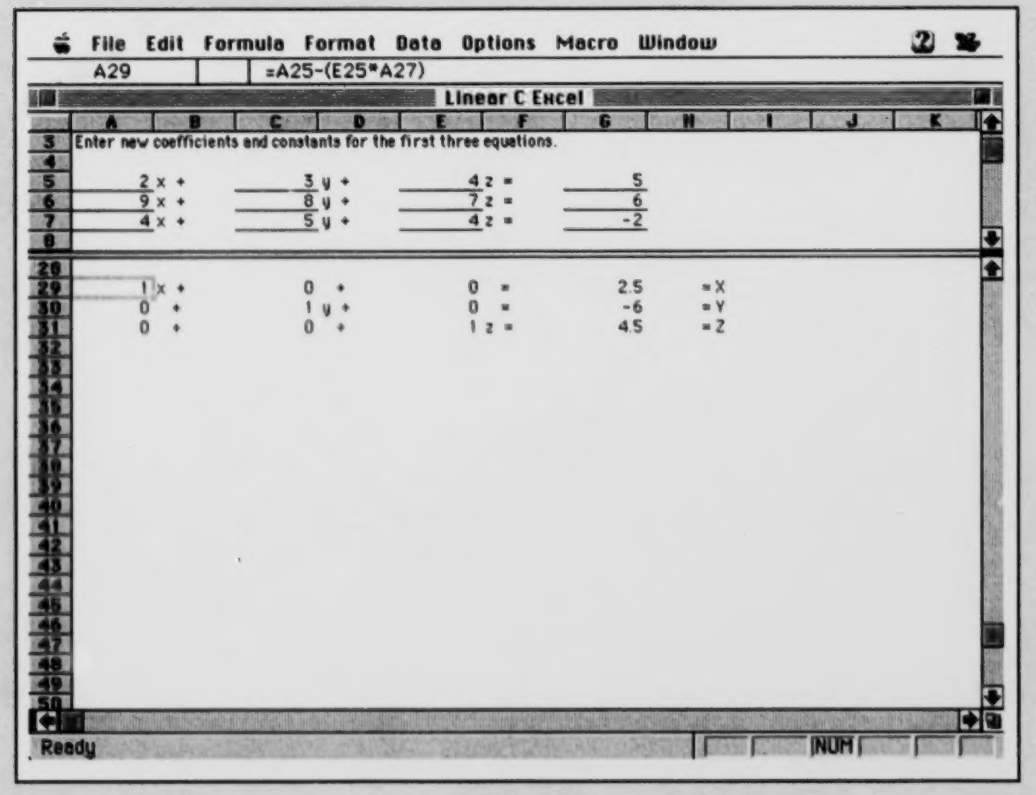
File Edit Formula Format Data Options Macro Window
A29
=A25-(E25*A27)
Linear C Excel
Enter new coefficients and constants for the first three equations.
2 x + 3 y + 4 z = 5
9 x + 8 y + 7 z = 6
4 x + 5 y + 4 z = -2
1 x + 0 + 0 = 2.5 =X
0 + 1 y + 0 = -6 =Y
0 + 0 + 1 z = 4.5 =Z
Ready
NUM

Fig. 15

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TOPIC: Volume

SPREADSHEET FILE: Volume Excel (Macintosh and Windows)
Volume Works (Macintosh and Windows)
Volume SYLK (Macintosh and Windows)

PURPOSE: To illustrate the use of the spreadsheet as a tool for problem solving

CONTENT OR GRADE LEVEL: Algebra through precalculus

DESCRIPTION: This file displays a word problem involving the volume of a box made by cutting out small squares from each corner of a given large paper square (fig. 16). The initial values given are for a square of paper 12 units on a side and an initial small square of 1 unit on a side to be cut from each corner of the 12-unit square. The initial value for the incremental increase in the size of the small squares to be cut is 0.1.

CELL FORMULAS:

A12 =D8	D12 =A12*(D7-(2*A12))*(D7-(2*A12))
A13 =A12+D9	D13 =A13*(D7-(2*A13))*(D7-(2*A13))
A14 =A13+D9	D14 =A14*(D7-(2*A12))*(D7-(2*A12))
...	...

EXPLORATION IDEAS FOR STUDENTS:

1. The original size of a square of paper is 12 units on a side. What is the volume of the resulting box if a 1-unit square is cut from each corner of this 12-unit square and the sides are folded up at right angles to the base?
2. If we cut a 1.5-unit square from each corner of the 12-unit square of paper, what will be the volume of the box?
3. What happens if we cut a 6-unit square from each corner of the 12-unit square of paper?
4. What is the maximum volume for a box made from the 12-unit square piece of paper?
5. What is the maximum volume for a box made from a 10-unit square piece of paper? Experiment with different values for the increment.
6. What if you had a square piece of cardboard 16 inches on a side and you wanted to make a box that would hold exactly 300 cubic inches of sand? What size square would you have to cut out from each corner of the cardboard to create this box?
7. Determine the maximum volume of boxes obtained from different sizes of large paper squares. Vary the increment for the sizes of the small squares to be cut from the corners.
8. Suppose you had a rectangular piece of paper, say, a units by b units. What formula could you use to help you find the maximum volume of a box obtained by cutting out squares of x units on a side from each corner? Use this formula to develop a spreadsheet file similar to this one.
9. Find a good problem and try to develop a spreadsheet file that would help solve that problem!

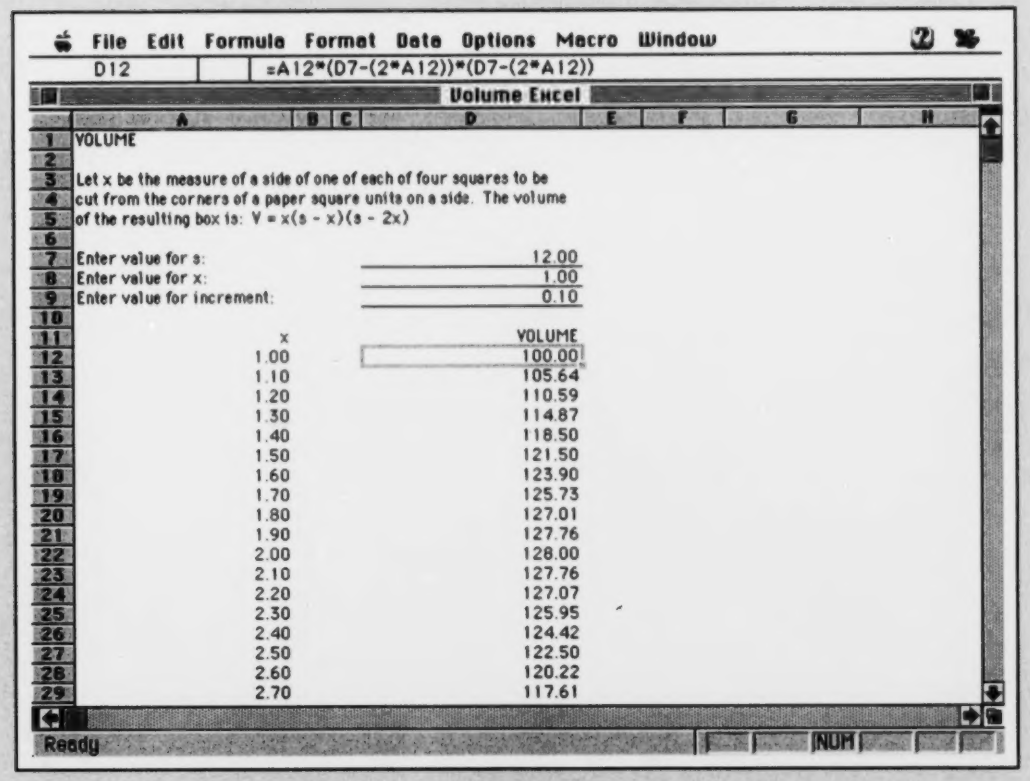

Fig. 16

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TOPIC: Mixture

SPREADSHEET FILE: Mixture Excel (Macintosh and Windows)
Mixture Works (Macintosh and Windows)
Mixture SYLK (Macintosh and Windows)

PURPOSE: To illustrate the use of the spreadsheet as a tool for problem solving

CONTENT OR GRADE LEVEL: Algebra through precalculus

DESCRIPTION: This file displays a word problem involving determining the cost per pound for various mixtures of two different types of coffees (fig. 17).

CELL FORMULAS:

	C8 =1-B8
B11 =B8*B9	C11 =C8*C9
	C13 =B11+C11

EXPLORATION IDEAS FOR STUDENTS:

1. Examine this file and change the percent for coffee A. How does the percent chosen for coffee A determine the percent used for coffee B?
2. Without using this file, predict what the final cost would be for 5 pounds of a 50-50 blend of the two coffees if coffee A costs $5.00 a pound and coffee B costs $4.00 a pound. What if you wanted a 60-40 blend at the same costs for each? Use the file to check your work.
3. If the cost for the blend is $4.00 a pound and coffee A costs $3.80 a pound, what percent of coffee A is used in the blend if the cost of coffee B is $4.20 a pound?
4. If the cost for the blend is $4.10 a pound and coffee A costs $4.00 a pound, what percent of coffee A is used in the blend if the cost of coffee B is $5.00 a pound?
5. Use this file as a guide to create other spreadsheet files that consider problems of this type.

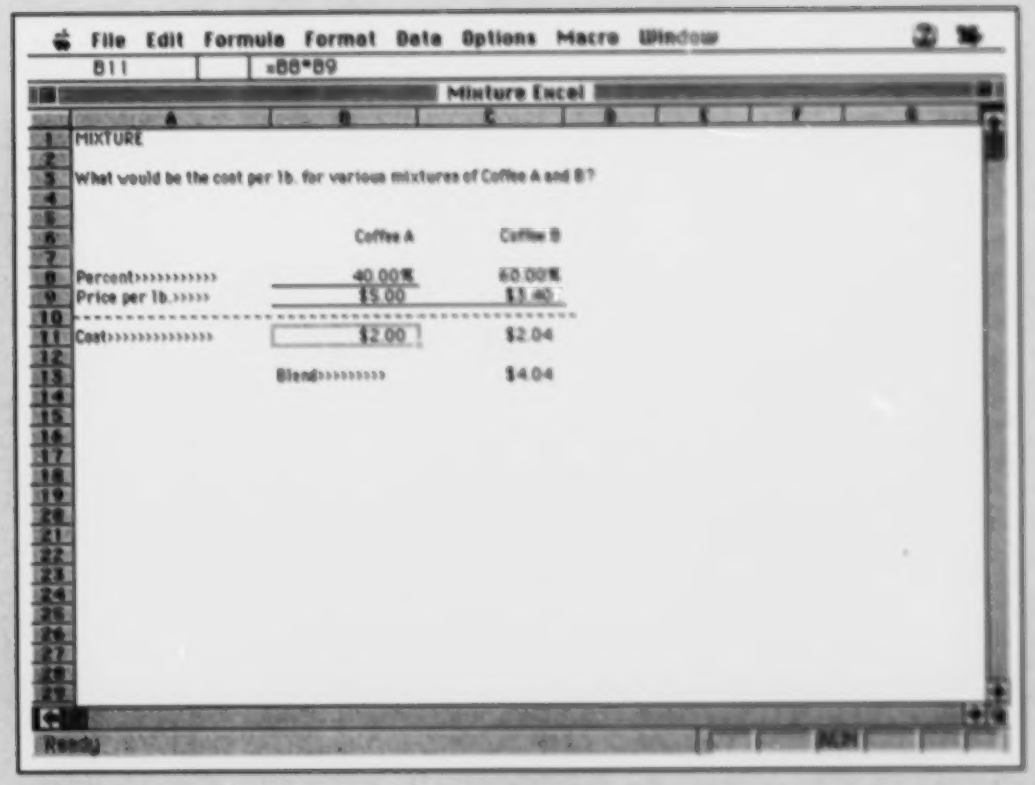

Fig. 17

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TOPIC: Completing the Square

SPREADSHEET FILE: Completing the Square Excel (Macintosh and Windows)
Completing the Square Works (Macintosh and Windows)
Completing the Square SYLK (Macintosh and Windows)

PURPOSE: To explore the method of completing the square to solve a quadratic equation

CONTENT OR GRADE LEVEL: Algebra through precalculus

DESCRIPTION: This file asks the user to

"Enter new coefficients and constant for the equation below:"

The equation displayed is

2x^2 + 4x + -6 = 0.

The file then proceeds, through a series of steps, to simplify and solve the equation by completing the square. A graph is constructed using the information in the block of cells A31:B71 to illustrate the line determined by the equation. (See fig. 18.)

CELL FORMULAS:

A7	=A5	C7	=C5/A5			G7	=-E5	
A9	=A5	C9	=C7	E9	=(.5*C9)^2	G9	=E5	I9 =A5*E9
A11	=A5	C11	=0.5*C9			G11	=-E5+I9	
		C13	=0.5*C9			G13	=G11/A11	
		C15	=0.5*C9			G15	=SQRT(G13)	I15 =-SQRT(G13)
						G17	=G15-C15	I17 =I15-C15

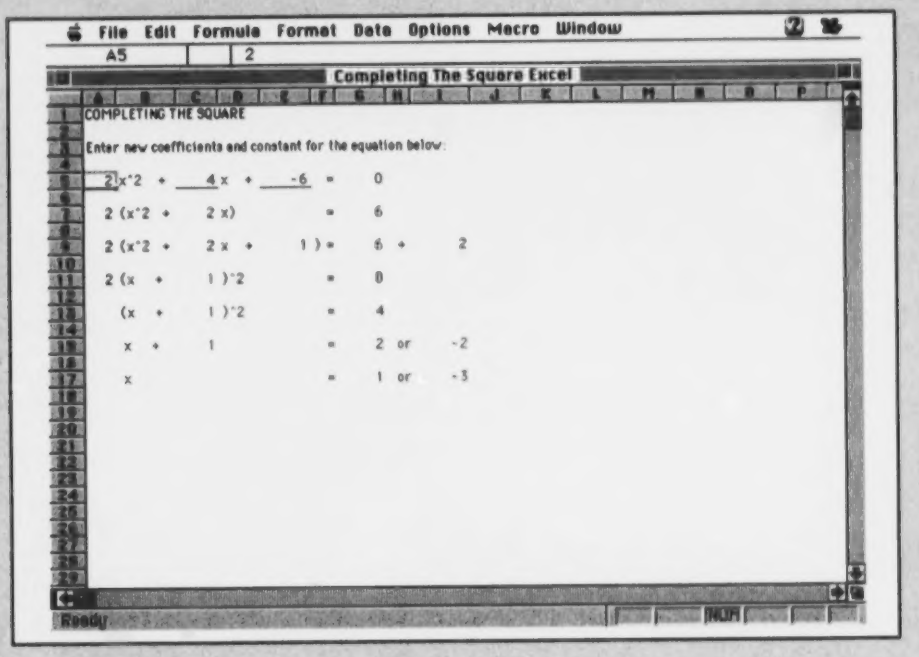
A31	-10	B31	=$A$5*A31^2+$C$5*A31+$E$5
A32	=A31+0.5	B32	=$A$5*A32^2+$C$5*A32+$E$5
...		...	
A71	=A70+0.5	B71	=$A$5*A71^2+$C$5*A71+$E$5

EXPLORATION IDEAS FOR STUDENTS:

For each of lines 1 through 6 below, refer to the file as it is set up with the equation illustrated above.

1. What is the constant term in row 5? The equation in row 5 is transformed into the equation in row 7 by subtracting the constant term, -6, from both sides of the equation. Then, the coefficient of the *x*-squared term is factored from the expression on the left side of the equal sign. What *is* the coefficient of the *x*-squared term?
2. What is 1/2 the coefficient of the *x* term in row 7? The equation in row 7 is transformed into the equation in row 9 by squaring 1/2 of the coefficient of the *x* term and adding it within the parentheses—to become part of the expression that is multiplied by the coefficient of the original *x* term. In order to maintain the balance on both sides of the equation, the product of these two numbers is added to the right side of the equation.
3. How is the equation in row 11 transformed from the equation in row 9?
4. How is the equation in row 13 transformed from the equation in row 11?
5. How is the equation in row 15 transformed from the equation in row 13?
6. How is the equation in row 17 transformed from the equation in row 15?
7. What happens to the solution if you change the constant term to 2?
8. What happens to the solution if you change the constant term to 5?

9. Try changing the constant term to numbers between 2 and 5. To greater than 5. What happens?
10. Try changing the constant term to numbers between 2 and -6. To less than -6. What happens?
11. Explore what happens as you try different values for the coefficient of the *x* term. Of the *x*-squared term.
12. Will this algorithm always work? If so, why? If not, why not?


File Edit Formula Format Data Options Macro Window
A5
2
Completing The Square Excel
COMPLETING THE SQUARE
Enter new coefficients and constant for the equation below:
2x^2 + 4x + -6 = 0
2 (x^2 + 2 x) = 6
2 (x^2 + 2 x + 1) = 6 + 2
2 (x + 1)^2 = 8
(x + 1)^2 = 4
x + 1 = 2 or -2
x = 1 or -3
Ready

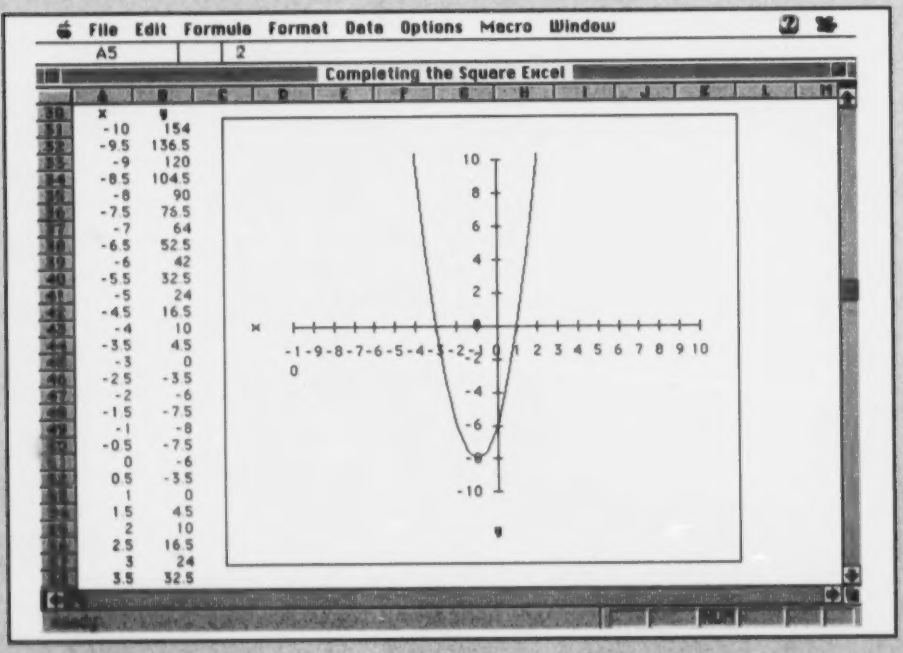


Fig. 18

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TOPIC: Synthetic Division and Integral Roots

SPREADSHEET FILE: Synthetic Division A Excel (Macintosh and Windows)
Synthetic Division A Works (Macintosh and Windows)
Synthetic Division A SYLK (Macintosh and Windows)

PURPOSE: To explore the method of synthetic division to solve a polynomial function for its integral roots

CONTENT OR GRADE LEVEL: Intermediate algebra through precalculus

DESCRIPTION: This file asks the user to

"Enter new coefficients and constant for the equation below:"

The equation displayed is

0x^3 + 1x^2 + 2x + -3 = 0.

Using the equation above as an example, row 11 asks the user to test each p where p is a factor of the constant term. On entering the requested number and pressing <return>, the file uses the standard procedure for synthetic division to test the number. If the number is a root, a zero will appear in cell G16, the "remainder cell." If the number being tested is not a root, the remainder will appear in cell G16. A graph is constructed using the information in the block of cells A31:B71 to illustrate the plot of the equation. (See fig. 19.)

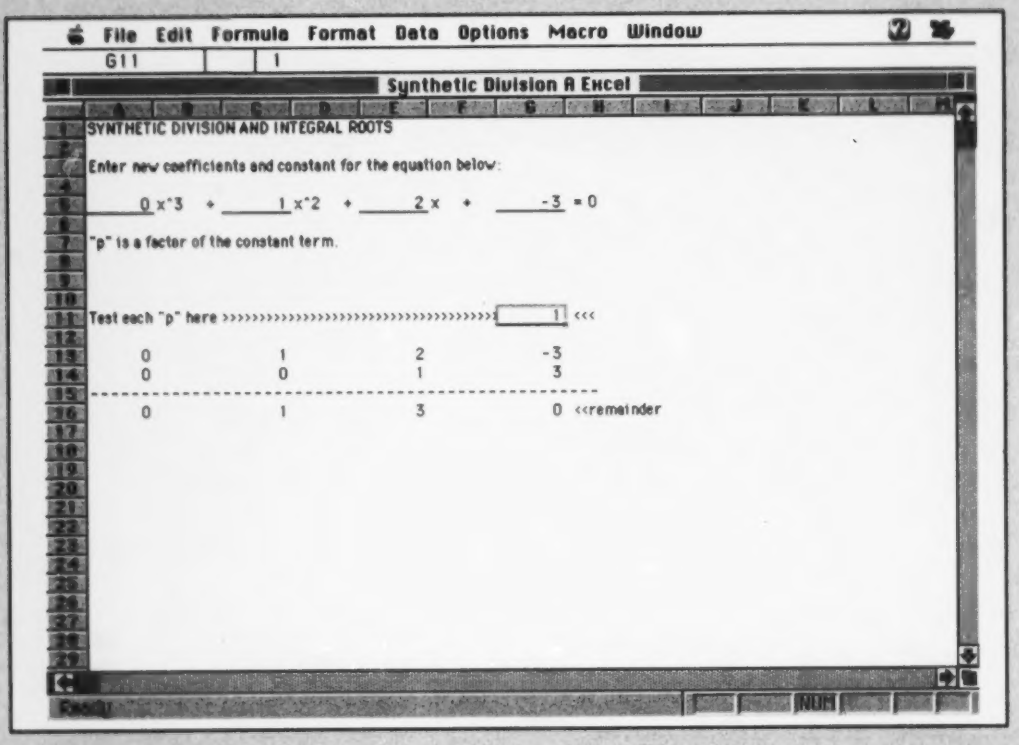
CELL FORMULAS:

A13 =A5	C13 =C5	E13 =E5	G13 =G5
A14 0	C14 =G11+A16	E14 =G11*C16	G14 =G11*E16
A16 =A13+A14	C16 =C13+C14	E16 =E13+E14	G16 =G13+G14
A31 -10	B31 =$A$5*A31^3+$C$5*A31^2+$E$5*A31+$G$5		
A32 =A31+.5	B32 =$A$5*A32^3+$C$5*A32^2+$E$5*A32+$G$5		
...	...		
A71 =A70+.5	B71 =$A$5*A71^3+$C$5*A71^2+$E$5*A71+$G$5		

EXPLORATION IDEAS FOR STUDENTS:

1. List all factors of the constant term, or p's, for the given equation and test each to determine if it is an integral root of the equation.
2. If we test all the factors of the constant term, why can we be sure that we have tested for all possible integral roots of the equation in question?
3. Enter new coefficients and constant for the equation at the top of the screen to obtain the following, and follow the indicated procedure to determine its integral roots:

   2x^3 - 9x^2 + 3x + 4 = 0
4. How does this file work? Study it along with your textbook to see that it uses the standard procedure for synthetic division and the integral roots theorem, except that it does most of the arithmetic for you.
5. Use this file to help you find integral roots to other second- and third-degree equations.


File Edit Formula Format Data Options Macro Window
G11
1
Synthetic Division A Excel
SYNTHETIC DIVISION AND INTEGRAL ROOTS
Enter new coefficients and constant for the equation below:
0 x^3 + 1 x^2 + 2 x + -3 = 0
"p" is a factor of the constant term.
Test each "p" here >>>>

TOPIC: Synthetic Division and Rational Roots

SPREADSHEET FILE: Synthetic Division B Excel (Macintosh and Windows)
Synthetic Division B Works (Macintosh and Windows)
Synthetic Division B SYLK (Macintosh and Windows)

PURPOSE: To explore the method of synthetic division to solve a polynomial function

CONTENT OR GRADE LEVEL: Intermediate algebra through precalculus

DESCRIPTION: This file asks the user to

"Enter new coefficients and constant for the equation below:"

The equation displayed is

2x^3 + 1x^2 + 5x + -3 = 0.

Using the equation above as an example, row 11 asks the user to test each p/q where p is a factor of the constant term and q is a factor of the first nonzero coefficient—in the example above, the coefficient of the x squared term. On entering the requested number and pressing <return>, the file uses the standard procedure for synthetic division to test the number. If the number is a root, a zero will appear in cell G16, the "remainder cell." If the number being tested is not a root, the remainder will appear in cell G16. A graph is constructed using the information in the block of cells A31:B71 to illustrate the plot of the equation. (See fig. 20.)

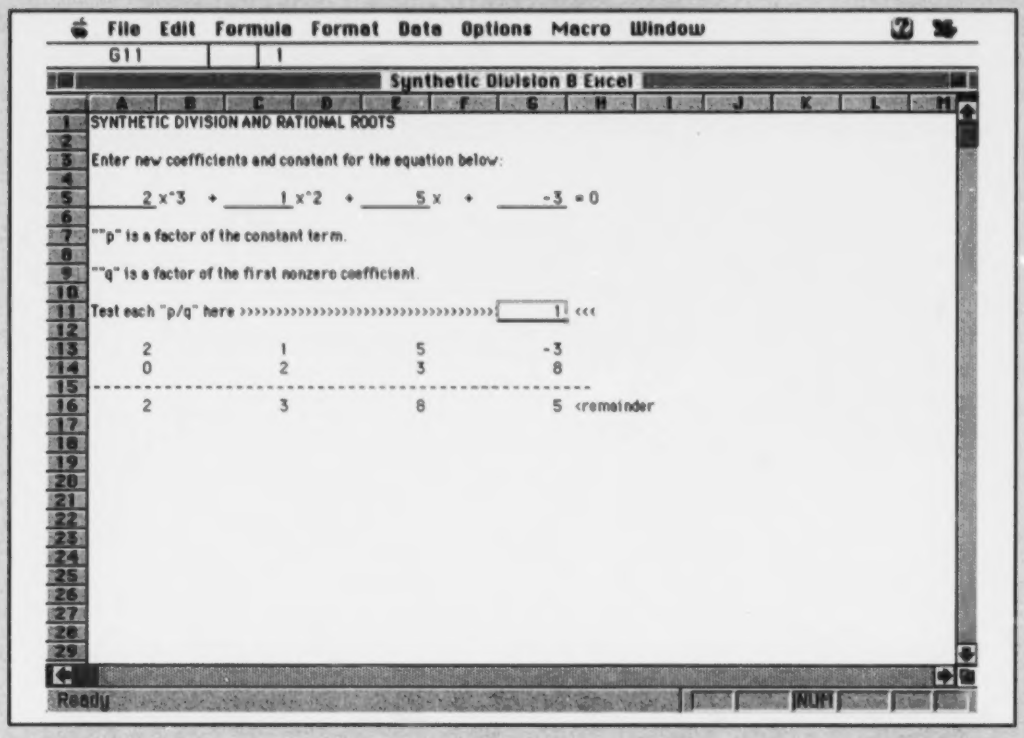
CELL FORMULAS:

A13 =A5	C13 =C5	E13 =E5	G13 =G5
A14 0	C14 =G11*A16	E14 =G11*C16	G14 =G11*E16
A16 =A13+A14	C16 =C13+C14	E16 =E13+E14	G16 =G13+G14
A31 -10	B31 =$A$5*A31^3+$C$5*A31^2+$E$5*A31+$G$5		
A32 =A31+0.5	B32 =$A$5*A32^3+$C$5*A32^2+$E$5*A32+$G$5		
...	...		
A71 =A70+0.5	B71 =$A$5*A71^3+$C$5*A71^2+$E$5*A71+$G$5		

EXPLORATION IDEAS FOR STUDENTS:

1. List all the p's and q's for the given equation and test each p/q to determine if it is a root of the equation.
2. If we test all the factors of the constant term, why can we be sure that we have tested for all possible integral roots of the equation in question?
3. Enter new coefficients and constant for the equation at the top of the screen to obtain the following, and follow the indicated procedure to determine its rational roots:

   2x^3 +1x^2 -10x -5 = 0
4. How does this file work? Study it along with your textbook to see that it uses the standard procedure for synthetic division and the rational roots theorem, except that it does most of the arithmetic for you.
5. Use this file to help you find roots to other second- and third-degree equations.


File Edit Formula Format Data Options Macro Window
G11
1
Synthetic Division B Excel
SYNTHETIC DIVISION AND RATIONAL ROOTS
Enter new coefficients and constant for the equation below:
2 x^3 + 1 x^2 + 5 x + -3 = 0
"p" is a factor of the constant term.
"q" is a factor of the first nonzero coefficient.
Test each "p/q" here >>>>

TOPIC: Roots by Successive Approximation

SPREADSHEET FILE: Roots by Successive A Excel (Macintosh and Windows)
Roots by Successive A Works (Macintosh and Windows
Roots by Successive A SYLK (Macintosh and Windows)

PURPOSE: To explore an algorithm to solve a polynomial function by the method of roots by successive approximation

CONTENT OR GRADE LEVEL: Intermediate algebra through precalculus

DESCRIPTION: This file displays the function

f(x) = 1x^4 + 2x^3 + -2x^2 + -6x^1 + -3

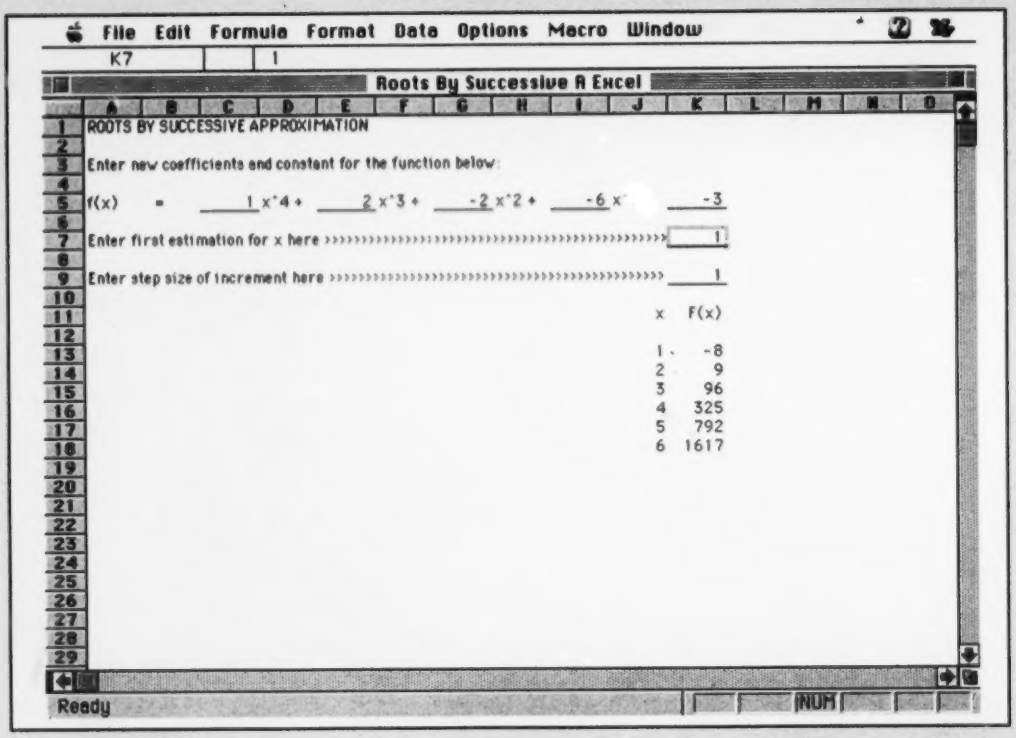
and asks the user to enter new coefficients and constant for it (fig. 21). The user is then asked to enter a first estimation for x and a step size for the increment to be used in the approximation process. For each new value of x or change in size of increment, the file will automatically substitute into the function and give corresponding values of $f(x)$. We are searching for values of x for which $f(x)$ is 0. A graph is constructed using the information in the block of cells A31:B71 to illustrate the plot of the function.

CELL FORMULAS:

J13	=K7	K13	=(C5•(J13^4))+(E5•(J13^3))+(G5•(J13^2))+(I5•(J13^1))+K5
J14	=J13+K9	K14	=(C5•(J14^4))+(E5•(J14^3))+(G5•(J14^2))+(I5•(J14^1))+K5
J15	=J14+K9	K15	=(C5•(J15^4))+(E5•(J15^3))+(G5•(J15^2))+(I5•(J15^1))+K5
J16	=J15+K9	K16	=(C5•(J16^4))+(E5•(J16^3))+(G5•(J16^2))+(I5•(J16^1))+K5
J17	=J16+K9	K17	=(C5•(J17^4))+(E5•(J17^3))+(G5•(J17^2))+(I5•(J17^1))+K5
J18	=J17+K9	K18	=(C5•(J18^4))+(E5•(J18^3))+(G5•(J18^2))+(I5•(J18^1))+K5
...		...	
A31	-10	B31	=$C$5•A31^4+$E$5•A31^3+$G$5•A31^2+$I$5•A31+$K$5
A32	=A31+0.5	B32	=$C$5•A32^4+$E$5•A32^3+$G$5•A32^2+$I$5•A32+$K$5
...		...	
A71	=A70+0.5	B71	=$C$5•A71^4+$E$5•A71^3+$G$5•A71^2+$I$5•A71+$K$5

EXPLORATION IDEAS FOR STUDENTS:

1. Using the example given in the file, we see that one zero for the function will be obtained when x has a value somewhere between 1 and 2, since for those values $f(x)$ switches signs—in this instance from negative to positive. Try by selecting smaller increments or new "first" estimates for x to obtain one or more values for x that will result in $f(x)$ being 0. Remember, we know that a value of x for which $f(x)$ is 0 is between two values that cause the values for $f(x)$ to change sign.
2. Use this file to solve other functions of the fourth power or less.
3. As set up, this file makes six substitutions for the value of x for each new "first estimation" of x or each new step size. Add as many new rows in the file for substitution as you would like. One word of caution—each new row will add to the time needed for the file to recalculate.


File Edit Formula Format Data Options Macro Window
K7
1
Roots By Successive A Excel
ROOTS BY SUCCESSIVE APPROXIMATION
Enter new coefficients and constant for the function below:
f(x) = 1 x^4 + 2 x^3 + -2 x^2 + -6 x -3
Enter first estimation for x here >>>>

TOPIC: Bisection Algorithm

SPREADSHEET FILE: Bisection Algorithm Excel (Macintosh and Windows)
Bisection Algorithm Works (Macintosh and Windows)
Bisection Algorithm SYLK (Macintosh and Windows)

PURPOSE: To explore the use of the bisection algorithm (interval halving) to solve a polynomial function

CONTENT OR GRADE LEVEL: Intermediate algebra through precalculus

DESCRIPTION: This file displays the function

f(x) = 1x^4 + 2x^3 + -2x^2 + -6x^1 + -3

and asks the user to enter new coefficients and constant for it. The user is then asked to enter new first interval endpoints, *P* and *Q*. After entering a 0 in cell M11 and pressing the appropriate key(s) displayed on the screen to reset the file, the user enters any nonzero number in cell M11 to start the bisection algorithm process. For each new calculation, the user presses the appropriate key(s) displayed on the screen. Calculations should continue until $f(P)$, $f(Q)$, and $f(M)$ become 0 and the new *P*, new *Q*, and new midpoint *M* become the same. (See fig. 22.)

In brief, the bisection algorithm works as follows: First, an interval *PQ* is determined on the *x*-axis such that $f(P)$ has a negative value and $f(Q)$ a positive value. If the function is continuous between the two points, then a zero of the function $f(x)$ is contained between them. The process locates the midpoint *M* of the interval and determines whether $f(M)$ is negative or positive. It then selects that half of the interval *PMQ* in which the endpoints have opposite signs. If $f(M)$ is negative, then *PM* is discarded and the search continues in the interval *MQ*—that is, the value of *P* is replaced by the value of *M*. If $f(M)$ is positive, then the interval *MQ* is discarded and the search continues in the interval *PM*—that is, the value of *Q* is replaced by the value of *M*. The process continues, each time selecting a new midpoint of the new and smaller interval, until endpoints and midpoint coincide to locate a root of the function. (A similar argument can be made for the case in which $f(P)$ has a positive value and $f(Q)$ a negative value.)

A graph is constructed using the information in the block of cells A31:B71 to illustrate the plot of the function.

CELL FORMULAS:

K9 =0.5*(C9+G9)
C15 =IF(M11=0,0,(C5*(C17^4))+(E5*(C17^3))+(G5*(C17^2))+(I5*C17)+K5)
G15 =IF(M11=0,0,(C5*(G17^4))+(E5*(G17^3))+(G5*(G17^2))+(I5*G17)+K5)
K15 =IF(M11=0,0,(C5*(K17^4))+(E5*(K17^3))+(G5*(K17^2))+(I5*K17)+K5)
C17 =IF(M11=0,C9,IF(C15*K15>0,K17,C17))
G17 =IF(M11=0,G9,IF(C17=K17,G17,K17))
K17 =0.5*(C17+G17)

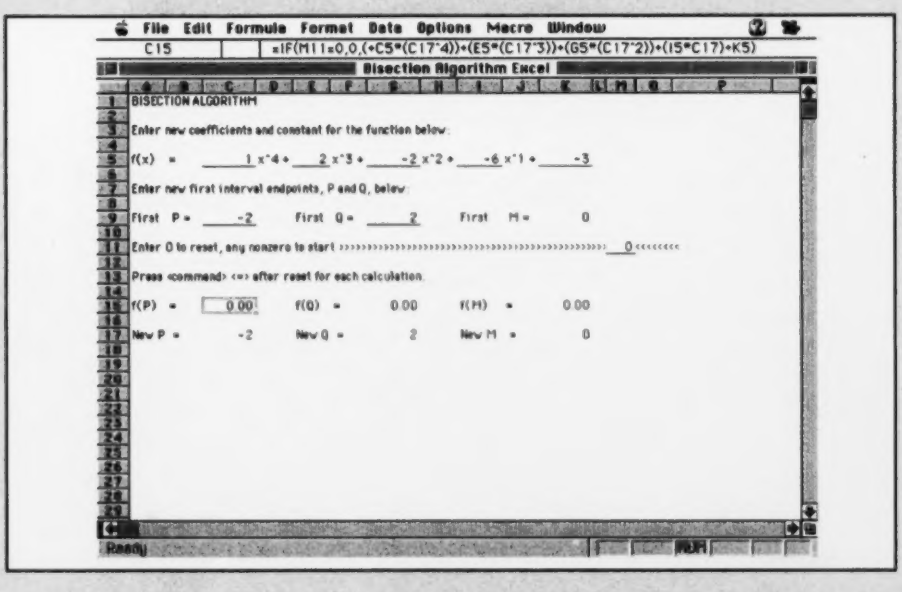
A31	-10	B31	=$C$5*A31^4+$E$5*A31^3+$G$5*A31^2+$I$5*A31+$K$5
A32	=A31+0.5	B32	=$C$5*A32^4+$E$5*A32^3+$G$5*A32^2+$I$5*A32+$K$5
...		...	
A71	=A70+0.5	B71	=$C$5*A71^4+$E$5*A71^3+$G$5*A71^2+$I$5*A71+$K$5

EXPLORATION IDEAS FOR STUDENTS:

Important: Always reset the file after changing coefficients, constant, and first *P* and *Q* by entering a 0 in

cell M11 and pressing the appropriate key(s) displayed on the screen. Do this *before* entering a nonzero number in cell M11.

1. Using the example given in the file and starting with a first P of -2 and a first Q of 2, the bisection algorithm quickly "zeros in" on -1.732 (the negative of the square root of 3, correct to three decimal places) for $f(P)$, $f(Q)$, and $f(M)$ as a root for this function. Try by selecting new first endpoints of the interval to obtain other roots of this function.
2. Will this algorithm always work? If so, why? If not, why not?
3. Use this file to explore other functions.


File Edit Formula Format Data Options Macro Window
C15
=IF(M11=0,0,(+C5*(C17^4))+(E5*(C17^3))+(G5*(C17^2))+(I5*C17)+K5)
Bisection Algorithm Excel
BISECTION ALGORITHM
Enter new coefficients and constant for the function below:
f(x) = 1 x^4 + 2 x^3 + -2 x^2 + -6 x^1 + -3
Enter new first interval endpoints, P and Q, below:
First P = -2 First Q = 2 First M = 0
Enter 0 to reset, any nonzero to start >>>>

TOPIC: Roots by Newton's Method of Approximation

SPREADSHEET FILE: Roots by Newton Excel (Macintosh and Windows)
Roots by Newton Works (Macintosh and Windows)
Roots by Newton SYLK (Macintosh)

PURPOSE: To explore an algorithm to solve a polynomial function by Newton's method of approximation

CONTENT OR GRADE LEVEL: Intermediate algebra through calculus

DESCRIPTION: This file displays the function

f(x) = 1x^4 + 2x^3 + -2x^2 + -6x^1 + -3

and requests the user to enter new coefficients and constant for it (fig. 23). The user is then asked to enter a first estimation for x. For each new value of x, the file automatically will use Newton's method of approximation to zero in on $f(x)$. Newton's method takes the first approximation of x and substitutes it in both $f(x)$ and $f'(x)$, the first derivative of $f(x)$. It then calculates a new approximation for x by subtracting from the last approximation of x the quotient of the last value for $f(x)$ by the last value for $f'(x)$. That is,

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

A graph is constructed using the information in the block of cells A31:B71 to illustrate the plot of the function.

CELL FORMULAS:

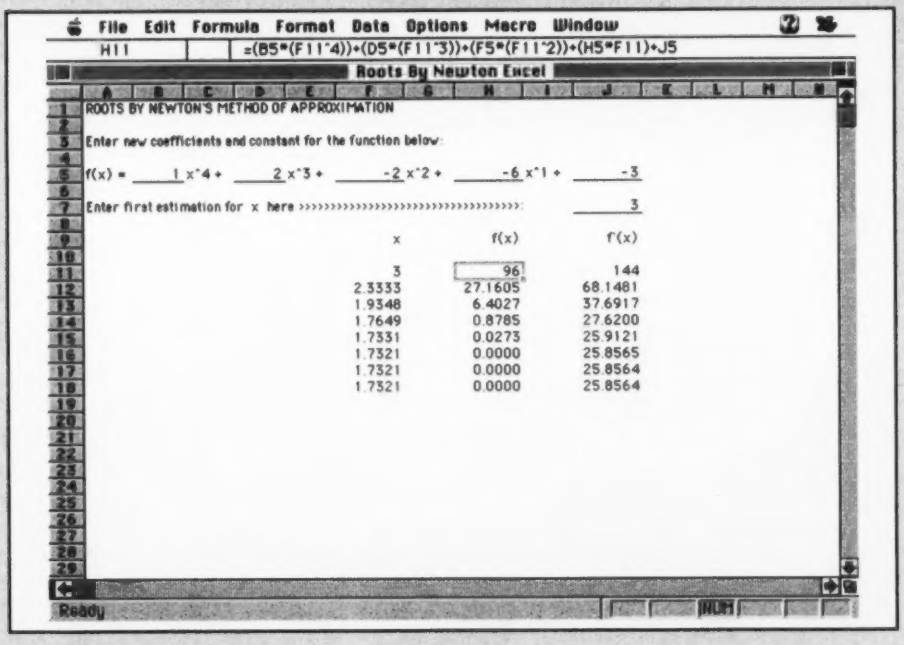
F11 =J7
H11 =(B5*(F11^4))+(D5*(F11^3))+(F5*(F11^2))+(H5*F11)+J5
J11 =((4*B5)*(F11^3))+((3*D5)*(F11^2))+((2*F5)*F11)+H5
F12 =F11-(H11/J11)
H12 =(B5*(F12^4))+(D5*(F12^3))+(F5*(F12^2))+(H5*F12)+J5
J12 =((4*B5)*(F12^3))+((3*D5)*(F12^2))+((2*F5)*F12)+H5
F13 =F12-(H12/J12)
H13 =(B5*(F13^4))+(D5*(F13^3))+(F5*(F13^2))+(H5*F13)+J5
J13 =((4*B5)*(F13^3))+((3*D5)*(F13^2))+((2*F5)*F13)+H5
F14 =F13-(H13/J13)
H14 =(B5*(F14^4))+(D5*(F14^3))+(F5*(F14^2))+(H5*F14)+J5
J14 =((4*B5)*(F14^3))+((3*D5)*(F14^2))+((2*F5)*F14)+H5
...

A31	-10	B31	=$B$5*A31^4+$D$5*A31^3+$F$5*A31^2+$H$5*A31+$J$5
A32	=A31+0.5	B32	=$B$5*A32^4+$D$5*A32^3+$F$5*A32^2+$H$5*A32+$J$5
...		...	
A71	=A70+0.5	B71	=$B$5*A71^4+$D$5*A71^3+$F$5*A71^2+$H$5*A71+$J$5

EXPLORATION IDEAS FOR STUDENTS:

1. Using the example given in the file, we see that one zero for the function $f(x)$ will be obtained with a first estimation for x of 3. The sixth approximation for x, 1.7320508 (an approximation for the square root of 3) leads to a value of 0 for $f(x)$. Try to determine as many of its other roots as you can.

2. Will this algorithm always work? If so, why? If not, why not?
3. Use this file to explore other functions of the fourth power or less.
4. As set up, this file makes six approximations for the value of x for each new first estimation of x. Add as many new rows in the file for substitution as you would like. One word of caution—each new row will add to the time needed for the file to recalculate.


File Edit Formula Format Data Options Macro Window
H11
=(B5*(F11^4))+(D5*(F11^3))+(F5*(F11^2))+(H5*F11)+J5
Roots By Newton Excel
ROOTS BY NEWTON'S METHOD OF APPROXIMATION
Enter new coefficients and constant for the function below:
f(x) = 1 x^4 + 2 x^3 + -2 x^2 + -6 x^1 + -3
Enter first estimation for x here >>>>

TOPIC: Determinants of Two Linear Equations with Two Variables

SPREADSHEET FILE: Determinants A Excel (Macintosh and Windows)
Determinants A Works (Macintosh and Windows)
Determinants A SYLK (Macintosh and Windows)

PURPOSE: To explore the method of solving systems of two linear equations with two variables by the use of determinants

CONTENT OR GRADE LEVEL: Intermediate algebra through precalculus

DESCRIPTION: This file displays two linear equations with two variables,

$$-1x + 2y = 4$$

$$1x + 6y = 8,$$

and asks the user to enter new coefficients and constant, c, for each one (fig. 24). The file then uses Cramer's rule, which uses determinants to solve the system of equations. In this file, the value for each determinant is calculated as follows:

determinant D is calculated as $x_1 y_2 - x_2 y_1$,
determinant D_x as $c_1 y_2 - c_2 y_1$, and
determinant D_y as $x_1 c_2 - x_2 c_1$.

Then,

x is calculated as D_x/D

and

y is calculated as D_y/D.

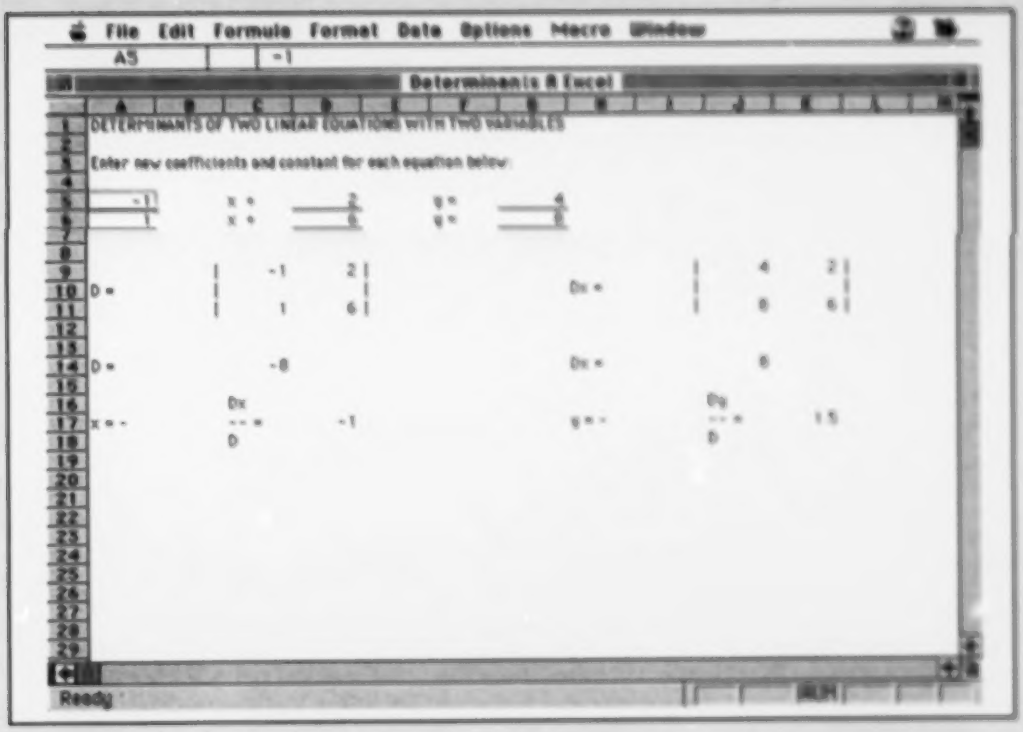
A graph is constructed using the information in the block of cells A31:C71 to illustrate the intersection of the lines determined by the two equations.

CELL FORMULAS:

C9	=A5	D9	=D5	J9	=G5	K9	=D5	Q9	=A5	R9	=G5
C11	=A6	D11	=D6	J11	=G6	K11	=D6	Q11	=A6	R11	=G6
C14	=(A5•D6)-(A6•D5)			J14	=(G5•D6)-(G6•D5)			Q14	=(A5•G6)-(A6•G5)		
		D17	=J14/C14			K17	=Q14/C14				
A31	-10	B31	=($A$5•A31-$G$5)/-$D$5			C31	=($A$6•A31-$G$6)/-$D$6				
A32	=A31+0.5	B32	=($A$5•A32-$G$5)/-$D$5			C32	=($A$6•A32-$G$6)/-$D$6				
...		...				...					
A71	=A70+0.5	B71	=($A$5•A71-$G$5)/-$D$5			C71	=($A$6•A71-$G$6)/-$D$6				

EXPLORATION IDEAS FOR STUDENTS:

1. This file uses Cramer's rule, which, in this illustration, uses two-by-two determinants to solve two equations in two variables. How are the determinants D, D_x, and D_y determined?
2. Study this file along with your textbook to develop an understanding of determinants and Cramer's rule.
3. Will this algorithm always work? If so, why? If not, why not?
4. Use this file to solve other systems of two equations with two variables.


File Edit Formula Format Data Options Macro Window
A5
-1
Determinants R Excel
DETERMINANTS OF TWO LINEAR EQUATIONS WITH TWO VARIABLES
Enter new coefficients and constant for each equation below:
Ready

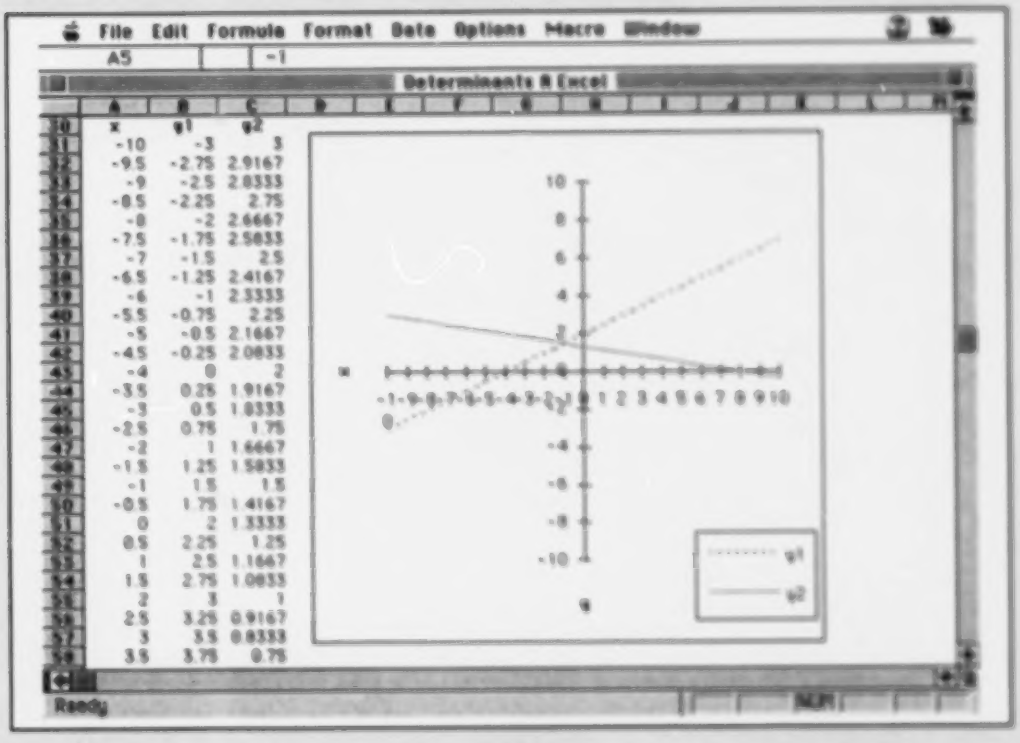


Fig. 24

BEST COPY AVAILABLE

TOPIC: Determinants of Three Linear Equations with Three Variables

SPREADSHEET FILE: Determinants B Excel (Macintosh and Windows)
Determinants B Works (Macintosh and Windows)
Determinants B SYLK (Macintosh and Windows)

PURPOSE: To explore the method of solving systems of three linear equations with three variables by the use of determinants

CONTENT OR GRADE LEVEL: Intermediate algebra through precalculus

DESCRIPTION: This file displays three linear equations with three variables,

$3x + -1y + 2z = 4$
$2x + 3y + -1z = 14$
$7x + -4y + 3z = -4,$

and asks the user to enter new coefficients and constant, c, for each one (fig. 25). It then uses Cramer's rule, which uses determinants to solve the system of equations. In this file, the value for each determinant is calculated by expanding by the minors of its top row. A minor is the two-by-two determinant that remains when the row and column containing the element of the top row are removed. This procedure is used to obtain the following:

$$D = x_1(y_2z_3 - y_3z_2) - y_1(x_2z_3 - x_3z_2) + z_1(x_2y_3 - x_3z_2)$$
$$D_x = c_1(y_2z_3 - y_3z_2) - y_1(c_2z_3 - x_3z_2) + z_1(c_2y_3 - x_3z_2)$$
$$D_y = x_1(c_2z_3 - c_3z_2) - c_1(x_2z_3 - x_3z_2) + z_1(x_2c_3 - x_3c_2)$$
$$D_z = x_1(y_2c_3 - y_3c_2) - y_1(x_2z_3 - x_3c_2) + c_1(x_2y_3 - x_3y_2)$$

Then,

x is calculated as D_x/D,

y is calculated as D_y/D, and

z is calculated as D_z/D.

CELL FORMULAS:

Q5 =A5	R5 =D5	S5 =G5
Q6 =A6	R6 =D6	S6 =G6
Q7 =A7	R7 =D7	S7 =G7
C9 =J5	D9 =D5	E9 =G5
C10 =J6	D10 =D6	E10 =G6
C11 =J7	D11 =D7	E11 =G7
J9 =A5	K9 =J5	L9 =G5
J10 =A6	K10 =J6	L10 =G6
J11 =A7	K11 =J7	L11 =G7
Q9 =A5	R9 =D5	S9 =J5
Q10 =A6	R10 =D6	S10 =J6
Q11 =A7	R11 =D7	S11 =J7

C13 =(A5•((D6•G7)-(D7•G6)))-(D5•((A6•G7)-(A7•G6)))+(G5•((A6•D7)-(A7•D6)))

H13 =(J5•((D6•G7)-(D7•G6)))-(D5•((J6•G7)-(J7•G6)))+(G5•((J6•D7)-(J7•D6)))

N13 =(A5•((J6•G7)-(J7•G6)))-(J5•((A6•G7)-(A7•G6)))+(G5•((A6•J7)-(A7•J6)))

S13 =(A5•((D6•J7)-(D7•J6)))-(D5•((A6•J7)-(A7•J6)))+(J5•((A6•D7)-(A7•D6)))

E16 =H13/C13

L16 =N13/C13

S16 =S13/C13

EXPLORATION IDEAS FOR STUDENTS:

1. This file uses Cramer's rule, which, in this illustration, uses three-by-three determinants to solve three equations in three variables. How are the determinants D, D_x, D_y, and D_z determined?
2. Study this file along with your textbook to develop an understanding of determinants and Cramer's rule.
3. Will this algorithm always work? If so, why? If not, why not?
4. Use this file to solve other systems of three equations with three variables.

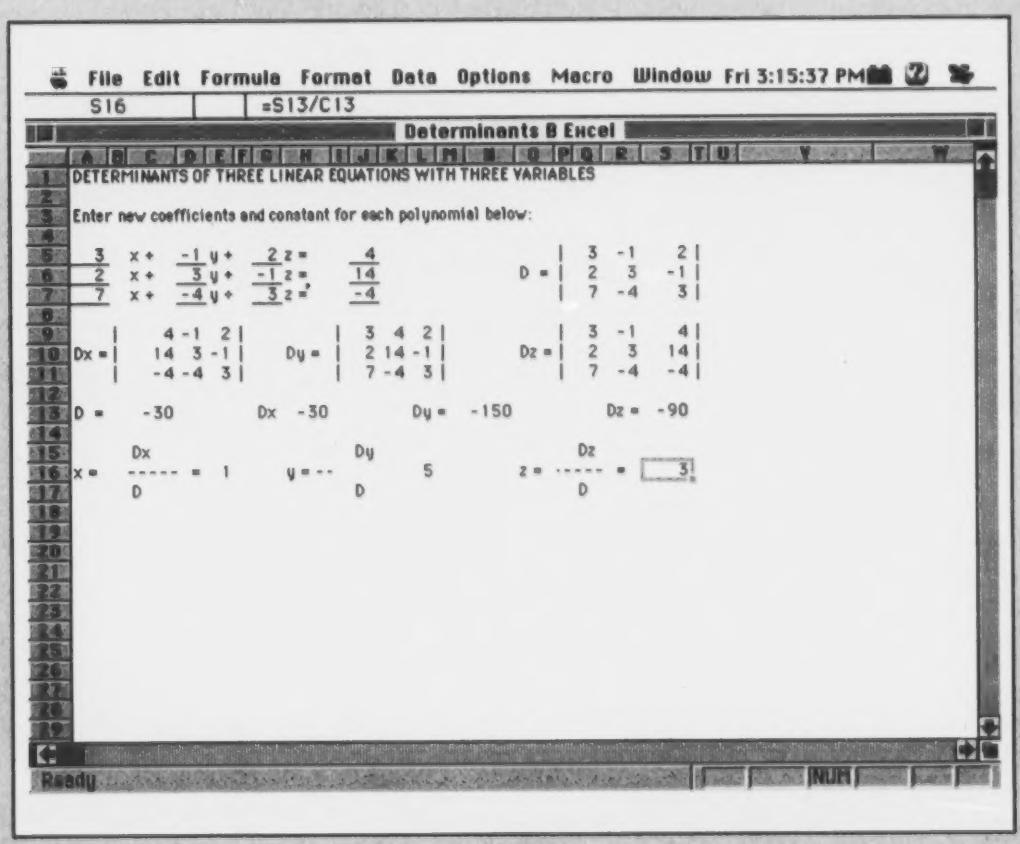

File Edit Formula Format Data Options Macro Window Fri 3:15:37 PM
S16
=S13/C13
Determinants B Excel
DETERMINANTS OF THREE LINEAR EQUATIONS WITH THREE VARIABLES
Enter new coefficients and constant for each polynomial below:
3 x + -1 y + 2 z = 4
2 x + 3 y + -1 z = 14
7 x + -4 y + 3 z = -4
D = | 3 -1 2 | | 2 3 -1 | | 7 -4 3 |
Dx = | 4 -1 2 | | 14 3 -1 | | -4 -4 3 |
Dy = | 3 4 2 | | 2 14 -1 | | 7 -4 3 |
Dz = | 3 -1 4 | | 2 3 14 | | 7 -4 -4 |
D = -30
Dx -30
Dy = -150
Dz = -90
x = Dx / D = 1
y = Dy / D = 5
z = Dz / D = 3
Ready
NUM

Fig. 25

BEST COPY AVAILABLE

TOPIC: Pascal's Triangle and Binomial Expansion

SPREADSHEET FILE: Pascal Triangle Excel (Macintosh and Windows)
Pascal Triangle Works (Macintosh and Windows)
Pascal Triangle SYLK (Macintosh and Windows)

PURPOSE: To explore the relationship of Pascal's triangle to the generation of terms in a binomial expansion

CONTENT OR GRADE LEVEL: Algebra 1 through precalculus

DESCRIPTION: This file displays two triangles (fig. 26). The first triangle displays rows one through six of Pascal's triangle in its usual form. The second triangle uses the numbers generated in the first triangle to indicate the coefficients of the expansion of the binomial to the left of each of the rows in the second triangle. The file is set up so that only the coefficients of the binomial in row 13 can be changed.

CELL FORMULAS:

L13 =L5•(B13)
K14 =K6•(B13^2)
J15 =J7•(B13^3)
I16 =I8•(B13^4)
H17 =H9•(B13^5)
N17 =N9•(B13^2)•(D13^3)
G18 =G10•(B13^6)
M18 =M10•(B13^3)•(D13^3)

N13 =N5•(D13)
M14 =M6•(B13•D13)
L15 =L7•(B13^2)•(D13^1)
P15 =P7•(D13^3)
K16 =K8•(B13^3)•(D13^1)
O16 =O8•(B13^1)*(D13^3)
J17 =J9•(B13^4)•(D13^1)
P17 =P9•(B13^1)•(D13^4)
I18 =I10•(B13^5)•(D13^1)
O18 =O10•(B13^2)•(D13^4)
S18 =S10•(D13^6)

O14 =O6•(D13^2)
N15 =N7•(B13^1)•(D13^2)
M16 =M8•(B13^2)•(D13^2)
Q16=Q8•(D13^4)
L17 =L9•(B13^3)•(D13^2)
R17 =R9•(B13^5)
K18 =K10•(B13^4)•(D13^2)
Q18 =Q10•(B13^1)•(D13^5)

EXPLORATION IDEAS FOR STUDENTS:

1. Examine each row of numbers in the top triangle of this file. How are they generated?
2. Change the coefficients of the binomial in row 13 to the following:

   (1x + 1y)^1

   What happens to the second triangle?
3. Change the coefficients in row 13 back to the following:

   (1x + -2y)^1

   Using paper and pencil, expand each of the displayed binomials and compare your results with the numbers indicated in the appropriate rows.
4. Examine each row of numbers in the bottom triangle of this file. How are they generated?
5. Change the coefficients in row 13 to create other binomials and examine their expansions as above.

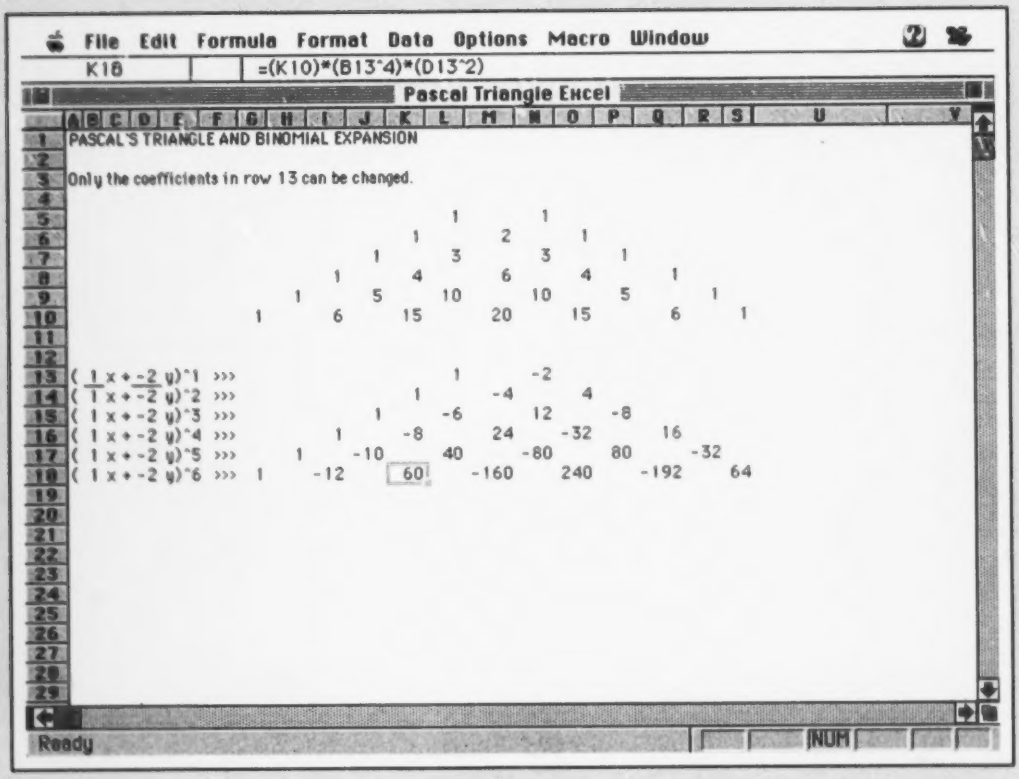

File Edit Formula Format Data Options Macro Window
K18
=(K10)*(B13^4)*(D13^2)
Pascal Triangle Excel
PASCAL'S TRIANGLE AND BINOMIAL EXPANSION
Only the coefficients in row 13 can be changed.
(1 x + -2 y)^1 >>>
(1 x + -2 y)^2 >>>
(1 x + -2 y)^3 >>>
(1 x + -2 y)^4 >>>
(1 x + -2 y)^5 >>>
(1 x + -2 y)^6 >>>
Ready
NUM

Fig. 26

BEST COPY AVAILABLE

TOPIC: Coin Toss

SPREADSHEET FILE: Coin Toss Excel (Macintosh and Windows)
Coin Toss Works (Macintosh and Windows)
Coin Toss SYLK (Macintosh and Windows)

PURPOSE: To explore the concept of probability through the simulated tossing of a single coin

CONTENT OR GRADE LEVEL: Grade 7 through intermediate algebra

DESCRIPTION: This file simulates the tossing of a single coin. In the example illustrated, the coin has been tossed four times, resulting in three heads (75%) and one tail (25%).

The file asks the user to enter 0 to reset or 1 to start. Then for each toss of the coin, the user must press the appropriate key(s) as displayed on the screen. The file then uses the random number function (RAND) to generate the numbers 1 and 2, determined randomly as needed, and indicated in cell C7. (See fig. 27.)

CELL FORMULAS:

C7 =IF(C3=0,0,INT(2*RAND()))

C9 =IF(C3=0,0,C9+1)

C11 =IF(C3=0,0,IF(C7=1,C11+1,C11))

C13 =IF(C3=0,0,C11/C9*100)

C15 =C9-C11

C17 =IF(C9=0,0,100-C13)

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file and use it to simulate the tossing of one coin 10 times. How many heads and how many tails result? What is the percentage for each?
2. Reset the file and toss the coin 100 times, again recording the number of heads and the number of tails and the percentage for each.
3. Reset the file and again toss the coin 100 times. Compare these results with the previous results.
4. If a coin is tossed 1000 times, what do you think the number of heads should be?
5. Examine each of the formulas in column C by placing the cursor on the appropriate cell. How are the numbers generated?
6. What generalizations can you make about tossing one coin?

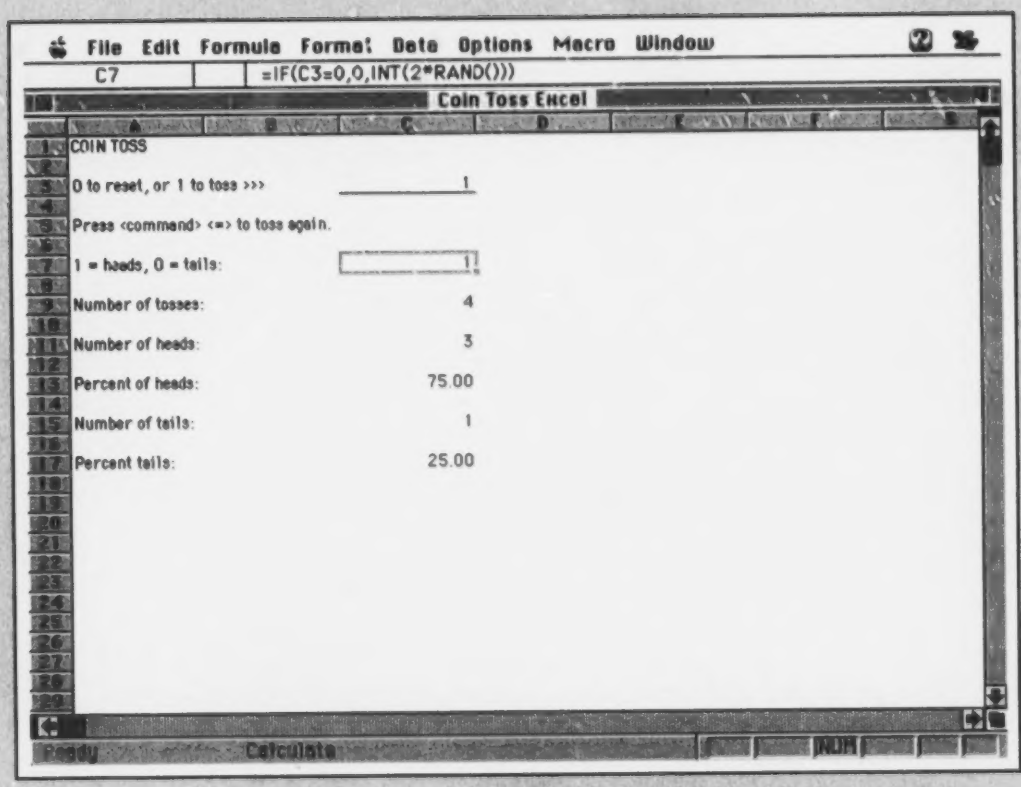


Fig. 27

BEST COPY AVAILABLE

TOPIC: Dice Toss

SPREADSHEET FILE: Dice Toss Excel (Macintosh and Windows)
Dice Toss Works (Macintosh and Windows)
Dice Toss SYLK (Macintosh and Windows)

PURPOSE: To explore the concept of probability through the simulated tossing of a pair of dice

CONTENT OR GRADE LEVEL: Grade 7 through intermediate algebra

DESCRIPTION: This file simulates the tossing of a pair of dice. In the example illustrated, the dice have been tossed ten times, resulting in zero 2s (0%), two 3s (20%), zero 4s (0%), one 5 (10%), zero 6s (0%), one 7 (10%), one 8 (10%), three 9s (30%), zero 10s (0%), one 11 (10%), and one 12 (10%).

The file asks the user to enter 0 to reset or 1 to start. Then, for each additional toss of the dice, the user must press the appropriate key(s) as displayed on the screen. The file then uses the RAND function to generate the numbers 1 through 12, determined randomly as needed, and indicated in cells A7 and C7. (See fig. 28.)

CELL FORMULAS:

A7	=IF(C3=0,0,INT(6*RAND()+1))	C7	=IF(C3=0,0,INT(6*RAND()+1)
A9	=IF(C3=0,0,A9+1)		
A11	=IF(C3=0,0,IF(A7+C7=2,A11+1,A11))	C11	=IF(C3=0,0,IF(A9=0,A11,100*A11/A9))
A12	=IF(C3=0,0,IF(A7+C7=3,A12+1,A12))	C12	=IF(C3=0,0,IF(A9=0,A12,100*A12/A9))
...		...	
A21	=IF(C3=0,0,IF(A7+C7=12,A21+1,A21))	C21	=IF(C3=0,0,IF(A9=0,A21,100*A21/A9))

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file and use it to simulate tossing the dice ten times. How many of each sum result? What is the percentage for each?
2. Reset the file and toss the dice 100 times, again recording the number of times each sum is obtained and the percentage for each.
3. Reset and again toss the dice 100 times. Compare these results with the previous results.
4. If the dice are tossed 1000 times, what do you think should be the number for each of the sums?
5. Examine each of the formulas in column C by placing the cursor on the appropriate cell. How are the numbers generated?
6. What generalizations can you make about tossing a pair of dice?

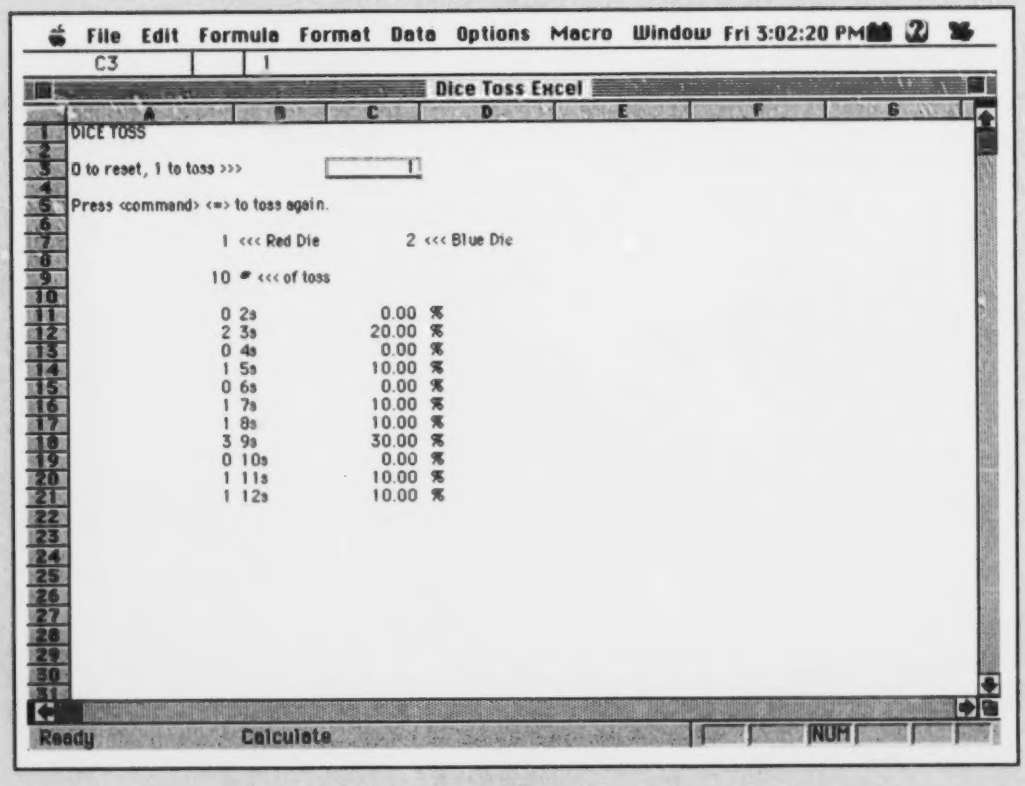


Fig. 28

BEST COPY AVAILABLE

TOPIC: Birth Month Match

SPREADSHEET FILE: Birth Month Match Excel (Macintosh and Windows)
Birth Month Match Works (Macintosh and Windows)
Birth Month Match SYLK (Macintosh and Windows)

PURPOSE: To explore the concept of probability through the simulated matching of the birth months of ten randomly selected people

CONTENT OR GRADE LEVEL: Grade 7 through intermediate algebra

DESCRIPTION: This file simulates the random selection of ten people and checks to see if at least two of them have the same birth month. In the example illustrated in figure 29, 25 such random selections resulted in 24 occurrences of at least two people in the group of ten having the same birth month.

CELL FORMULAS:

E7 =IF(C3=0,0,E7+1)

E9 =IF(C3=0,0,IF(SUM((C9:C18)>0,E9+1,E9)

E11 =IF(C3=0,0,E9/E7*100)

B9 =IF(C3=0,0,INT(12*RAND()+1))

B10 =IF(C3=0,0,INT(12*RAND()+1))

...

B18 =IF(C3=0,0,INT(12*RAND()+1))

C9 =IF(C3=0,0,IF(OR(B9=B10,B9=B11,B9=B12,B9=B13,B9=B14,B9=B15,
B9=B16,B9=B17,B9=B18)1,0))

C10 =IF(C3=0,0,IF(OR(B10=B9,B10=B11,B10=B12,B10=B13,B10=B14,B10=B15,
B10=B16,B10=B17,B10=B18)1,0))

...

C18 =IF(C3=0,0,IF(OR(B18=B9,B18=B10,B18=B11,B18=B12,B18=B13,B18=B14,
B18=B15,B18=B16,B18=B17)1,0))

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file and then enter the number 1 to indicate one trial. See if there are at least two persons in that group of ten with the same birth month. Are there other matches in this trial as well?
2. Reset the file and indicate 10 trials. Record the number of times that at least two people in a trial have the same birth month. What percent of the trials had matches?
3. Reset the file again and enter 100 trials. Compare these results with the previous results. Repeat this process several times.
4. Examine each of the formulas in columns B, C, and E by placing the cursor on the appropriate cell. How are the numbers generated?
5. What generalization can you make about the probability of at least two people in a group of ten having the same birth month?

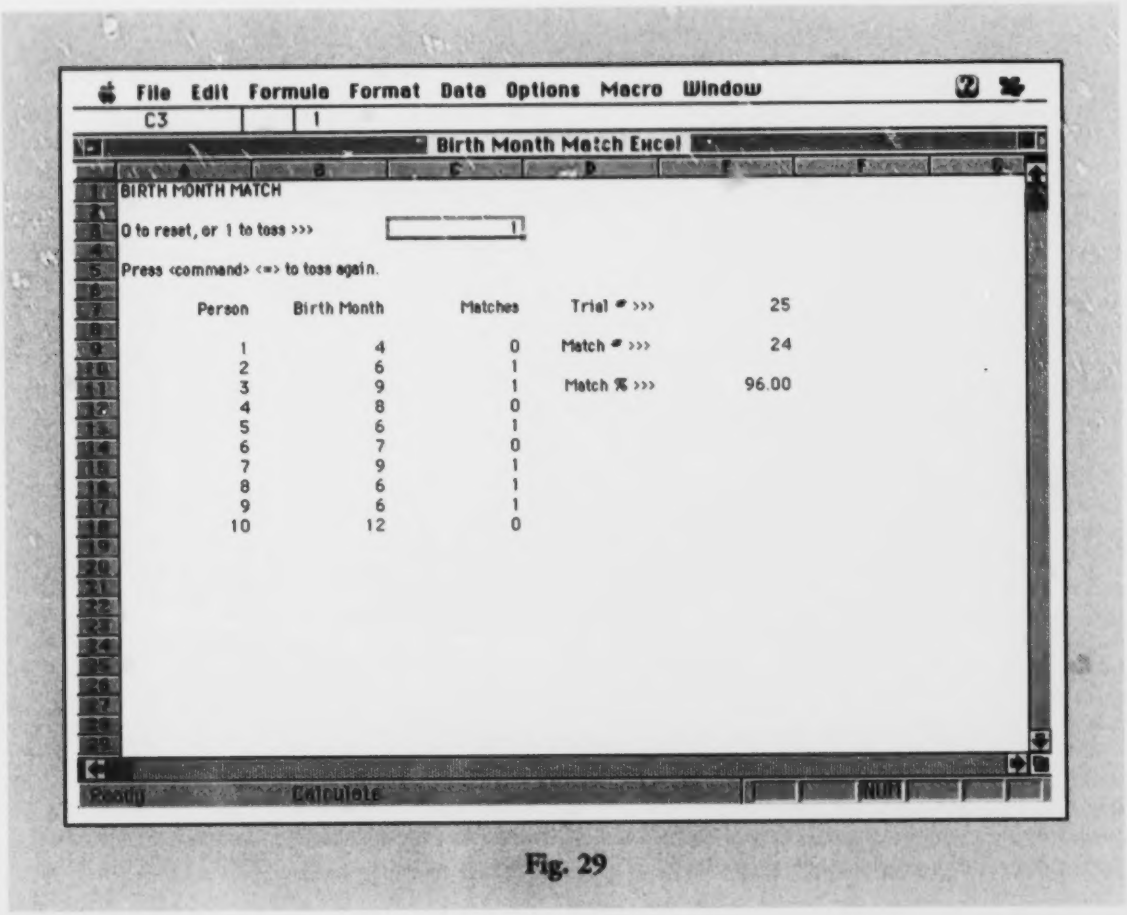


Fig. 29

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TOPIC: Birthday Match

SPREADSHEET FILE: Birthday Match Excel (Macintosh and Windows)
Birthday Match Works (Macintosh and Windows)
Birthday Match SYLK (Macintosh and Windows)

PURPOSE: To explore the concept of probability through the simulated matching of birthdays of ten randomly selected people

CONTENT OR GRADE LEVEL: Grade 7 through intermediate algebra

DESCRIPTION: This file simulates the random selection of ten people and checks to see if at least two of them have the same birthday. In the example illustrated, 25 such random selections resulted in 3 occurrences of at least two people in the group of ten having the same birthday.

The file asks the user to enter 0 to reset or the number of trials of ten people wanted to start. The file then uses the RAND function to generate the numbers 1 through 365 to indicate the days of the year, determined randomly as needed and indicated in cells B9 through B18 (see fig. 30).

CELL FORMULAS:

E7 =IF(C3=0,0,E7+1)

E9 =IF(C3=0,0,IF(SUM(C9:C18)>0,E9+1,E9))

E11 =IF(C3=0,0,E9/E7*100)

B9 =IF(C3=0,0,INT(365*RAND()+1))

B10 =IF(C3=0,0,INT(365*RAND()+1))

...

B18 =IF(C3=0,0,INT(365*RAND()+1))

C9 =IF(C3=0,0,IF(OR(B9=B10,B9=B11,B9=B12,B9=B13,B9=B14,B9=B15,
B9=B16,B9=B17,B9=B18)1,0))

C10 =IF(C3=0,0,IF(OR(B10=B9,B10=B11,B10=B12,B10=B13,B10=B14,B10=B15,
B10=B16,B10=B17,B10=B18)1,0))

...

C18 =IF(C3=0,0,IF(OR(B18=B9,B18=B10,B18=B11,B18=B12,B18=B13,B18=B14,
B18=B15,B18=B16,B18=B17)1,0))

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file and enter the number 1 to indicate one trial. See if there are at least two people in that group of ten with the same birthday.
2. Reset the file and indicate 10 trials. Record the number of times that at least two people in a trial have the same birthday. What percent of the trials had matches?
3. Reset the file again and enter 100 trials. Compare these results with the previous results.
4. Examine each of the formulas in columns B, C, and E by placing the cursor on the appropriate cell. How are the numbers generated?
5. What generalization can you make about the probability of at least two people in a group of ten having the same birthday?

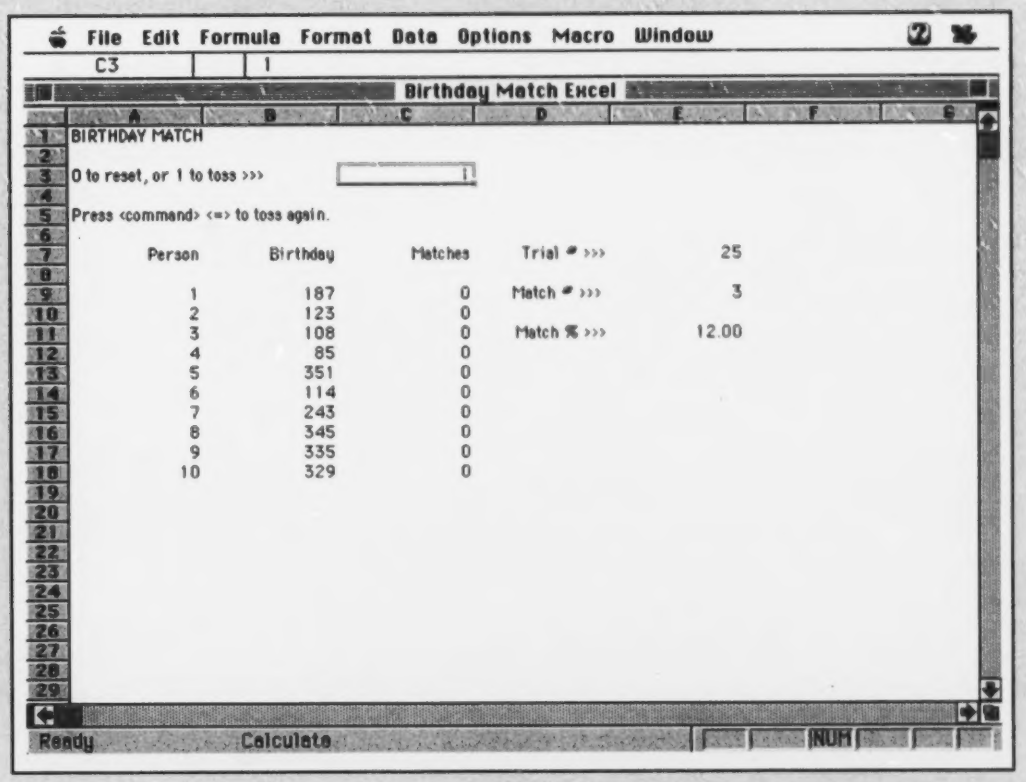

Fig. 30

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TOPIC: Card Draw with Replacement

SPREADSHEET FILE: Card Draw with Replace Excel (Macintosh and Windows)
Card Draw with Replace Works (Macintosh and Windows)
Card Draw with Replace SYLK (Macintosh and Windows)

PURPOSE: To explore the concept of probability through the simulated drawing of cards (with replacement) from a standard deck of fifty-two cards

CONTENT OR GRADE LEVEL: Grade 7 through intermediate algebra

DESCRIPTION: This file simulates the random selection of a card from a standard deck of fifty-two cards. As set up, the file replaces each card drawn, thus making it possible to select the same card more than once when making multiple draws.

In the example illustrated in figure 31, ten random selections resulted in the 4 of diamonds being drawn once, the 9 of diamonds being drawn once, the queen of diamonds being drawn once, the 3 of clubs being drawn once, the 7 of hearts being drawn once, the 8 of hearts being drawn once, the king of hearts being drawn once, the 5 of spades being drawn once, and the 6 of spades being drawn twice.

CELL FORMULAS:

B7 =IF(E3=0,0,INT(4*RAND()+1))
G7 =IF(E3=0,0,INT(13*RAND()+1))
B11 =IF(E3=0,0,IF(AND(B7=1,G7=2),B11+1,B11))
C11 =IF(E3=0,0,IF(AND(B7=1,G7=3),C11+1,C11))
...
N11 =IF(E3=0,0,IF(AND(B7=1,G7=1),N11+1,N11))
O11 =IF(E3=0,0,SUM(B11:N11))
B12 =IF(E3=0,0,IF(AND(B7=2,G7=2),B12+1,B12))
C12 =IF(E3=0,0,IF(AND(B7=2,G7=3),C12+1,C12))
...
N12 =IF(E3=0,0,IF(AND(B7=2,G7=1),N12+1,N12))
O12 =IF(E3=0,0,SUM(B12:N12))
...
B16 =SUM(B11:B14) ...
B18 =IF(E3=0,0,(B16/O16*100)) ...

O16 =SUM(O11:O14)
O18 =IF(E3=0,0,(O16*100))
O20 =IF(E3=0,0,(O11/(O16)*100))
...
O23 =IF(E3=0,0,(O14/(O16)*100))
O25 =SUM(O20:O23)

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file and enter the number 1 to draw. See what card is drawn randomly from the deck.
2. Keep pressing the appropriate key(s) as displayed on the screen until you have made 100 draws. Which cards were drawn? How many of each value and how many of each suit? Record your results. Repeat for another 100 draws. Compare these results with the previous results.

3. Keep pressing the appropriate key(s) as displayed on the screen until you have made 1000 draws. What generalizations can you make about the probability of drawing any card? Of drawing a spade? A 7?

4. Examine each of the formulas in the file by placing the cursor on the appropriate cell. How are the numbers generated?

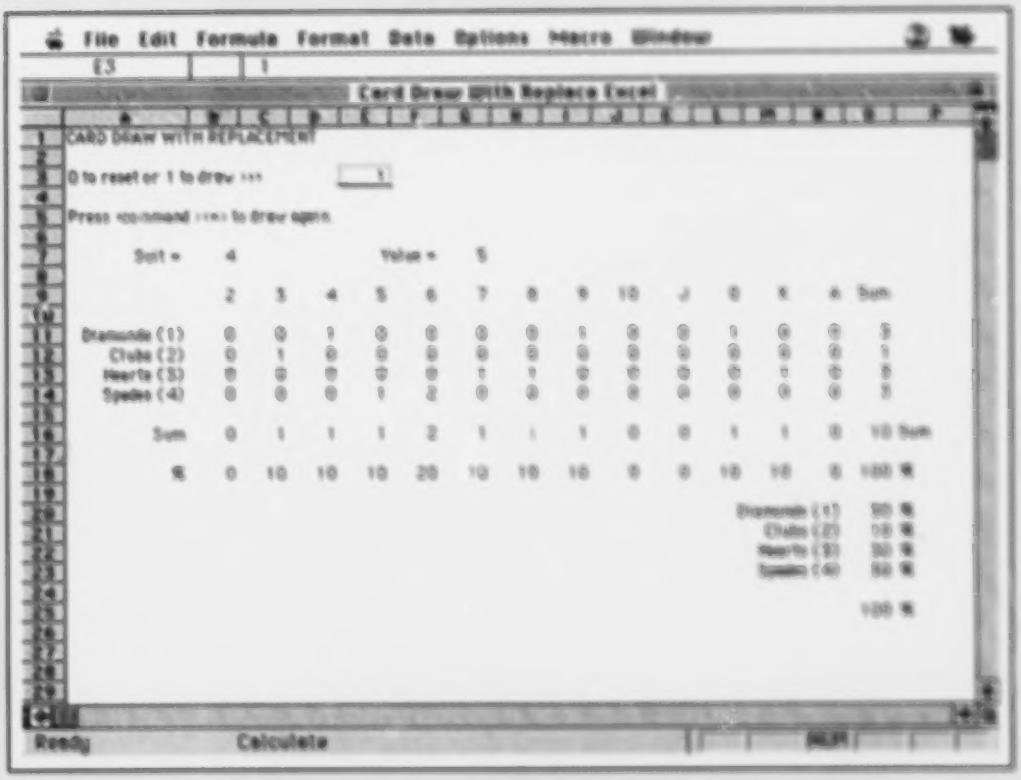

Fig. 31

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TOPIC: Card Draw without Replacement

SPREADSHEET FILE: Card Draw without Replac Excel(Macintosh and Windows)
Card Draw without Replac Works (Macintosh and Windows)
Card Draw without Replac SYLK (Macintosh and Windows)

PURPOSE: To explore the concepts of probability and combinations through the simulated drawing of cards (without replacement) from a standard deck of fifty-two cards

CONTENT OR GRADE LEVEL: Grade 7 through intermediate algebra

DESCRIPTION: As with the previous topic, Card Draw with Replacement, this file simulates the random selection of a card from a standard deck of fifty-two cards. Unlike the previous file, this file does not replace each card drawn.

In the example illustrated in figure 32, ten random selections resulted in each of the following cards being drawn: 3 of diamonds, 4 of diamonds, 3 of clubs, 4 of clubs, jack of clubs, 5 of hearts, 10 of hearts, 7 of spades, queen of spades, and king of spades.

The file asks the user to enter 0 to reset or 1 to make a draw. The file then uses the RAND function to generate the numbers 1 through 4 to indicate the suits and 1 through 13 to indicate the values, determined randomly as needed and indicated in cells B7 and G7.

CELL FORMULAS:

B7 =IF(E3=0,0,INT(4*RAND()+1))
G7 =IF(E3=0,0,INT(13*RAND()+1))
B11 =IF(E3=0,0,IF(AND(B7=1,G7=2),1,B11))
C11 =IF(E3=0,0,IF(AND(B7=1,G7=3),1,C11))
...
N11 =IF(E3=0,0,IF(AND(B7=1,G7=1),1,N11))
O11 =IF(E3=0,0,SUM(B11:N11))
B12 =IF(E3=0,0,IF(AND(B7=2,G7=2),1,B12))
C12 =IF(E3=0,0,IF(AND(B7=2,G7=3),1,C12))
...
N12 =IF(E3=0,0,IF(AND(B7=2,G7=1),1,N12))
O12 =IF(E3=0,0,SUM(B12:N12))
B16 =SUM(B11:B14) ...
B18 =IF(E3=0,0,(B16/O16*100)) ...

O16 =SUM(O11:O14)
O18 = IF(E3=0,0,(O16*100))
O20 =IF(E3=0,0,(O11/O16)*100))
...
O23 =IF(E3=0,0,(O14/O16)*100))
O25 =SUM(O20:O23)

EXPLORATION IDEAS FOR STUDENTS:

1. Reset the file and enter the number 1 to indicate one draw. See what card is drawn randomly from the deck.
2. Keep pressing the appropriate key(s) as displayed on the screen until you have made five draws. What

hand do you get? Record your results. Reset the file, repeat for another five draws, and record your results or your new five-card hand. Do this several times. Compare the results.

3. How many different ways are there to deal five cards from a standard deck of fifty-two cards if the order in which the cards are dealt *is* important? The solution to this problem involves the concept of permutations.

4. How many different ways are there to deal five cards from a standard deck of fifty-two cards if the order in which the cards are dealt *is not* important? The solution to this problem involves the concept of combinations.

5. Examine each of the formulas in the file by placing the cursor on the appropriate cell. How are the numbers generated?

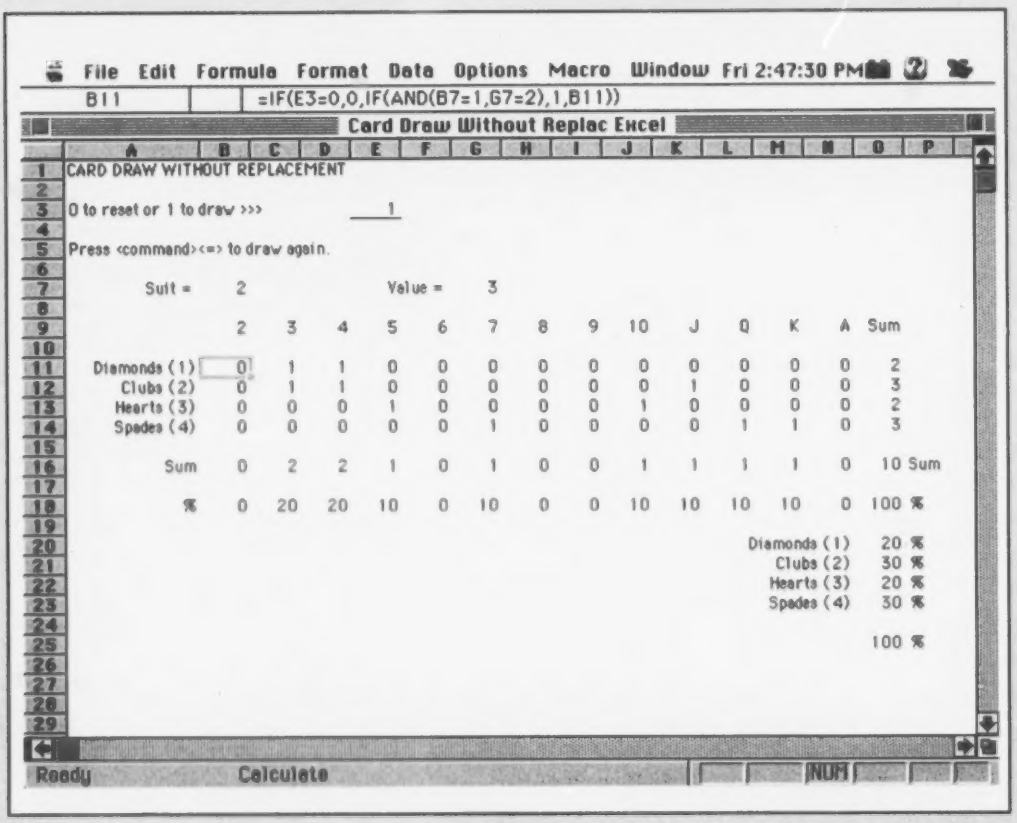
File Edit Formula Format Data Options Macro Window Fri 2:47:30 PM
B11
=IF(E3=0,0,IF(AND(B7=1,G7=2),1,B11))
Card Draw Without Replac Excel
CARD DRAW WITHOUT REPLACEMENT
0 to reset or 1 to draw >>>
1
Press <command><=> to draw again.
Suit = 2
Value = 3
2 3 4 5 6 7 8 9 10 J Q K A Sum
Diamonds (1) 0 1 1 0 0 0 0 0 0 0 0 0 0 2
Clubs (2) 0 1 1 0 0 0 0 0 0 1 0 0 0 3
Hearts (3) 0 0 0 1 0 0 0 0 1 0 0 0 0 2
Spades (4) 0 0 0 0 0 1 0 0 0 0 1 1 0 3
Sum 0 2 2 1 0 1 0 0 1 1 1 1 0 10 Sum
% 0 20 20 10 0 10 0 0 10 10 10 10 0 100 %
Diamonds (1) 20 %
Clubs (2) 30 %
Hearts (3) 20 %
Spades (4) 30 %
100 %
Ready
Calculate
NUM

Fig. 32

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ANSWER KEY

Magic Arrays (p. 6)

1. One correct arrangement is
   first row, 8, 1, 6; second row, 3, 5, 7; third row, 4, 9, 2.
2. Add any number to each of the above cells to generate a new magic array.
3. A new magic array is created.
4. A new magic array is created.
5. =SUM(B14,D10,F6). =SUM(B6,D10,F14). The solution of #1 above will work.
6. The same results will occur.
7. Extend the pattern of the three-by-three magic array. One correct arrangement for completion of a four-by-four magic array is the following:

   first row, 1, 15, 14, 4;
   second row, 12, 6, 7, 9;
   third row, 8, 10, 11, 5;
   fourth row, 13, 3, 2, 16.

   Extend the pattern of the four-by-four magic array. One correct arrangement for completion of a five-by-five magic array is the following:

   first row, 8, 1, 24, 17, 15;
   second row, 5, 23, 16, 14, 7;
   third row, 22, 20, 13, 6, 4;
   fourth row, 19, 12, 10, 3, 21;
   fifth row, 11, 9, 2, 25, 18.

Collecting and Analyzing Data A (p. 8)

The answers to the questions above will vary according to the particular students and age level of your class. You can expect to observe, however, that the older the children are, the more closely the average height will match the average reach of ten or more students.

Collecting and Analyzing Data B (p. 10)

The answers to the questions above will vary according to the opinions of the students who participate in the survey.

Collecting and Analyzing Data C (p. 12)

The answers to the questions above will vary.

Square Roots (p. 14)

1. The number in each row of the quotient column is obtained by dividing the original *S* by the average of the previous divisor and quotient.
2. Estimate was too small. Estimate was too large.
3. 5.
4. 6.
6. The best guess would be the actual square root of the number, but the divide-and-average method

quickly averages in on the square root in just a few trials if the first estimate is somewhere in the general area of the square root. Since 3 000 would be too small and 4 000 would be too large ($3\,000^2$ = 9 000 000 and $4\,000^2$ = 16 000 000), a good first estimation for the square root of 12 345 678 would be between the two, say 3 500.

7. Only one if the first estimate is actually the square root of the number.
8. Be as careful as possible with the first estimation. See answer to question 6 above.

Grade Book (p. 16)

1. Betty; 4; 80
2. 89
3. 73
4. 78.40
5. 74. To use the spreadsheet to determine the average, enter the formula =AVERAGE(B9:F9) in cell G9.
6. =SUM will sum the numbers listed from the first indicated cell to the last indicated cell.
   =COUNT will count the numbers listed from the first indicated cell to the last indicated cell.
   Each formula determines the quotient of the sum by the count in rows 14 and 15 above it.
   =MIN selects the minimum value from the first indicated cell to the last indicated cell.
   =MAX selects the maximum value from the first indicated cell to the last indicated cell.
   =AVERAGE determines the average of the numbers listed from the first indicated cell to the last indicated cell.

Multiples and Prime Numbers A (p. 18)

1. Each multiple of the number 3 is replaced in the file with a 0. They are all multiples of the number 3.
2. They are all multiples of the number 2.
4. Since the number 2 already blanked out all the multiples of 2, entering the numbers 4 and 8 does not blank out any new numbers—all multiples of 4 and 8 are also multiples of 2. Since 6 is a multiple of 2 and 3, entering the number 6 does not blank out any other numbers. Since 9 is a multiple of 3, entering the number 9 does not blank out any numbers. Since 10 is a multiple of 2 (and also of 5), it does not blank out any other numbers.
5. The number 11 is the last number to blank out numbers in the array.
6. Since 140 is the last number in the array, we can stop entering numbers after we have tried 11—12 is the square root of 144 and all multiples of 12 less than 144 have already been blanked as multiples of numbers less than 12.
7. The resulting nonblanked cells are all the prime numbers less than 141.

Multiples and Prime Numbers B (p. 20)

1. They are all multiples of the number 2.
2. They are all multiples of the number 3.
3. Except for the numbers 2 and 3, all numbers are either one more or one less than some multiple of 6.
4. All pairs of twin primes except 3 and 5 are separated by a multiple of 6.
5. 3,5; 5,7; 11,13; 17,19; 29,31; 41,43; 59,61; 71,73

Compound Interest A (p. 22)

1. $100 will double in value by the end of the twelfth year ($201.2196). $38.50 will double in value by

the end of the twelfth year ($77.4696). In fact, any investment at an annual rate of 6.0 percent interest compounded annually will double in value in twelve years.

2. $100 will double in value by the end of the twelfth year ($203.2794).
3. At 6 percent interest it is impossible to double the value by the end of the eleventh year, no matter how many times a year the interest is compounded. Refer to the spreadsheet file, Interest B, to examine this in more detail.
4. At 12.25 percent interest compounded annually, it would double in value by the end of six years ($200.0406).
5. 0.0825 percent.

Compound Interest B (p. 24)

1. Yearly, 6 percent; monthly, 6.17 percent; daily, 6.18 percent; every .01 second, 6.18 percent
2. The greater the number of times a per year, the greater the annual yield
3. $19,396.71($21,335.2810 compounded annually and $21,435.2810 compounded daily)
4. $(1.025)^4$

Linear Equations with One Equation and One Unknown (p. 26)

1. In row 7 of the spreadsheet, the constant term on the left side of the equation is subtracted from both sides of the equation to produce $2x + 4 - 4 = 10 - 4$.
2. In row 9, the indicated subtraction of the equation above is completed to produce $2x = 6$.
3. In row 11, both sides of the equation are divided by the coefficient of the x term to produce $2x / 2 = 6 / 2$.
4. In row 13, the indicated division of the equation above is completed to produce the solution, $x = 3$.
6. First, the constant term is subtracted from both sides of the equation. Second, both sides of the equation are divided by the coefficient of the x term.
8. Since it is always possible to subtract the constant from both sides of the equation and, assuming that the coefficient of the x term is not 0, to divide both sides of the equation by the coefficient of the x term, the algorithm always will work.

Linear Equations with Two Equations and Two Unknowns (p. 28)

1. Both sides of the equation are divided by 2, which is the coefficient of the x term.
2. The equation in row 8 is multiplied by the coefficient of the x term of the equation in row 9, and the resulting equation is subtracted from the equation in row 9.
3. The 0 is ignored, and the resulting equation is divided by the coefficient of the y term.
4. The equation in row 14 is multiplied by the coefficient of the y term of the equation in row 15, and the resulting equation is subtracted from the equation in row 15.
6. Both sides of the first equation are divided by the coefficient of its x term. Next, the new version of the first equation is multiplied by the coefficient of the x term of the second equation and subtracted from the second equation. Then, both sides of the new version of the second equation are divided by the coefficient of its y term. Finally, the latest version of the second equation is multiplied by the coefficient of the y term of the latest version of the first equation and subtracted from the latest version of the first equation.
8. This algorithm will always work to produce the solution to the system of two linear equations with two unknowns *as long as their graphs are not parallel lines or their graphs are not the same.* If the graphs are

parallel lines, there is no common solution to the system. If the graphs are the same, there are an infinite number of solutions. In either case, error messages will appear in various parts of the equations on the screen.

Linear Equations with Three Equations and Three Unknowns (p. 30)

3. a) Both sides of the first equation are divided by the coefficient of its x term.
   b) The new version of the first equation is multiplied by the coefficient of the x term of the second equation and subtracted from the second equation.
   c) The new version of the first equation is multiplied by the coefficient of the x term of the third equation and subtracted from the third equation.
   d) Both sides of the new version of the second equation are divided by the coefficient of its y term.
   e) The latest version of the second equation is multiplied by the coefficient of the y term of the latest version of the first equation and subtracted from the latest version of the first equation.
   f) The latest version of the second equation is multiplied by the coefficient of the y term of the latest version of the third equation and subtracted from the latest version of the third equation.
   g) Both sides of the new version of the third equation are divided by the coefficient of its y term.
   h) The latest version of the third equation is multiplied by the coefficient of the z term of the latest version of the first equation and subtracted from the latest version of the first equation.
   i) The latest version of the third equation is multiplied by the coefficient of the z term of the latest version of the second equation and subtracted from the latest version of the second equation.
4. This algorithm will always work to produce the solution to the system of three linear equations with three unknowns *as long as the graphs of any two are not parallel or skew lines and the graphs of any two are not the same.* In each case, error messages will appear in various parts of the equations on the screen.

Volume (p. 32)

1. 100 cubic units
2. 121.5 cubic units
3. We obtain a volume of 0 cubic units. Note that as we continue to increase the size of the square beyond 6 units on a side, the volume appears to increase. Of course this is not possible since squares larger than 6 units on a side cannot be cut from a sheet of paper 12 units on a side! However, this inconsistency with reality is a good departure point for discussion about modeling reality.
4. 128 cubic units
5. 74.07 cubic units
6. A square 3 inches on a side to produce a maximum volume of 300 cubic inches
8. $V = x(a - x)(b - x)$

Mixture (p. 34)

1. Whatever percent is selected for coffee A in cell B8, it is subtracted from 1 in cell C8 to determine the percent for coffee B.
2. The cost per pound would be \$4.50, so 5 pounds would cost \$22.50. The cost per pound of a 60-40 blend would be \$4.60, so 5 pounds would cost \$23.00.
3. 50 percent
4. 90 percent

Completing the Square (p. 36)

1. The constant term is –6. The coefficient of the x term is 2
2. Half the coefficient of the x term is 1.
3. On the left side of the equals sign the expression within the parentheses is indicated as the square of its square root and the expression on the right side of the equals sign is simplified.
4. Both sides of the equation are divided by the original coefficient of the x-squared term.
5. The square root of both sides of the equation is taken.
6. The constant term 1 is subtracted from both sides of the equation.
7. There is only one rational root, –1.
8. Error messages appear that indicate that there are no rational roots.
9. Error messages continue to appear, indicating that there are no rational roots for these equations.
10. Each equation will have exactly two rational roots.
12. Two conditions must exist for this algorithm to work. First, there must be a nonzero coefficient of the x-squared term. Second, assuming that the first condition is met, if an error message is indicated in line 15, there are no real roots to the equation. The complex roots can be calculated by hand using the information in line 13.

Synthetic Division and Integral Roots (p. 38)

1. The factors, or p's, of the constant term are –3, –1, 1, 3. The numbers 1 and –3 are integral roots of the equation.
2. By the integral roots theorem, any integral root of a polynomial must be a factor of the constant term.
3. The factors, or p's, of the constant term, or 4, are 4, 2, 1, –4, –2, –1. The numbers 4 and 1 are integral roots of the equation.
4. This file is a direct application of the synthetic division algorithm and the integral root theorem found in most intermediate algebra and precalculus textbooks.

Synthetic Division and Rational Roots (p. 40)

1. The factors, or p's, of the constant term are –3, –1, 1, 3. The factors of the first nonzero coefficient, or 2, are –2, –1, 1, 2. All possible p/q's are –3, –3/2, –1, –1/2, 1/2, 1, 3/2, 3. The number 1/2 is the only rational root of the equation.
2. By the integral roots theorem, any integral root of a polynomial must be a factor of the constant term.
3. The factors, or p's, of the constant term, or –5, are –5, –1, 1, 5. The factors of the first nonzero coefficient, or 2, are 2, 1, –2, –1. All possible p/q's are –5, –5/2, –1, –1/2, 1/2, 1, 5/2, 5. The number –1/2 is the only rational root of the equation.
4. This file is a direct application of the synthetic division algorithm found in most intermediate algebra and precalculus textbooks. Its use of p/q is a direct application of the rational roots theorem found in most precalculus textbooks.

Roots by Successive Approximation (p. 42)

1. One series of steps that will lead to a 0 for $f(x)$ is as follows: By changing the first estimation for x to 1.5, the screen will show that for $x = 1.5$, $f(x) = -4.6875$ and for $x = 2.5$, $f(x) = 39.8125$. By changing the step size of the increment to 0.1, the file will indicate that for $x = 1.7$, $f(x) = -.8019$ and for $x = 1.8$, $f(x) = 1.8816$. We can continue to zero in on a value of x for which $f(x) = 0$ and can quickly discover that at $x = 1.73205$ (an approximation for the square root of 3), the file will indicate that $f(x) = 0$.

By selecting a new first estimate of x as, for example, –5 and incrementing by 1, we will quickly determine that –1 also is a root of the equation, since that value of x results in 0 for $f(x)$ as well.

The third root, –1.73205, may prove to be a bit more elusive, since the graph of the function looks a bit like a "W" with the left "valley" formed by the curve crossing the x-axis at that point, descending to (–1.5, –1.875) and quickly ascending to cross the x-axis again at –1. Careful examination with a small step size of 0.01 will pay off, showing that the x-axis is crossed between values of –1.74 and –1.73.

Bisection Algorithm (p. 44)

1. The numbers –1 and 1.732 (the square root of 3, correct to three decimal places) also are roots for this function.
2. If the initial $f(P)$ and $f(Q)$ are opposite in sign or even if they are of the same sign but $f(M)$ is of the opposite sign, then the algorithm will always work. If either of these conditions exist, then we have trapped a zero of the function within the interval PM or MQ.

Roots by Newton's Method of Approximation (p. 46)

1. Other values of x that produce a value of 0 for $f(x)$ are –1 and –1.7320508.
2. This algorithm will not work if the first estimate of x is at a relative minimum point on the curve of the graph of the function, since $f'(x)$, the slope of the tangent line to the curve at this point, will be 0 and cause a division-by-zero error on the next approximation. In the example above, as estimate of –1 for x leads to an $f(x)$ of 0 and an $f'(x)$ of 0, the latter value indicating that the point (–1, 0) is either a relative maximum—as it is in this example—or a relative minimum. An estimate of 1 for x leads to an $f(x)$ of –8 and $f'(x)$ of 0, the latter value indicating that the point (1, –8) is either a relative minimum or a relative maximum—in this example, a relative minimum. Other values of x might cause successive approximations to diverge or become cyclic. Consult a calculus textbook for a more detailed discussion of this problem.

Determinants of Two Linear Equations with Two Variables (p. 48)

3. If the denominator determinant is zero and the numerator determinant is nonzero, the graphs of the equations do not intersect. In that case, the equations are inconsistent and have no solution. If both the numerator and denominator determinants are zero, then either the graphs of the equations are the same line or plane, or they are planes that either intersect in a line and have an infinite number of solutions (the equations are dependent) or do not intersect and thus have no solution.

Determinants of Three Linear Equations with Three Variables (p. 50)

3. If the denominator determinant is zero and the numerator determinant is nonzero, the graphs of the equations do not intersect. In that case, the equations are inconsistent and have no solution. If both numerator and denominator determinants are zero, then either the graphs of the equations are the same line or plane, or they are planes that either intersect in a line and have an infinite number of solutions (the equations are dependent) or do not intersect and thus have no solution.

Pascal's Triangle and Binomial Expansion (p. 52)

1. Each number is generated by adding the two numbers just above it in the previous row.
2. The second triangle becomes exactly like the first one.
4. Each number in any given row is the coefficient of the corresponding term of the binomial indicated at the left of the row. For example, the number in cell L18 is the *third* number in that row. It is the coefficient of the *third* term of the expansion of the binomial at the left. It is generated as the product of a) the number in the corresponding cell in Pascal's triangle above and b) the number which is the product

of the fourth power of the first coefficient in the binomial and the second power of the second coefficient in the binomial.

Coin Toss (p. 54)

1. Results will vary but should approximate 50 percent heads and 50 percent tails—if the coin is tossed a *large* number of times or if the average of the results of the entire class is considered.
2. See answer 1.
3. See answer 1.
4. See answer 1.
5. a) In C7, the function RAND generates a number between 0 and .999999999. 2*(RAND()) generates a number between 0 and just less than 2, such as 1.9999999. INT(2*(RAND()) generates the whole number portion of the number, either a 0 or a 1.

   b) In C9, if the relation C3=0 is true, then a 0 will be placed in cell C9. Otherwise, the value in C9 will be increased by 1.

   c) In C11, if the relation C3=0 is true, a 0 will be placed in C11. If the relation C7=1 is true, the value in C11 will be increased by 1. Otherwise the value in C11 will remain the same as it was.

   d) In C13, if the relation C3=0 is true, a 0 will be placed in C13. Otherwise, 100 times the quotient of the value in C11 divided by the value in C9 will be placed in C13.

   e) In C15, the difference between the value in C11 and the value in C9 is placed in C15.

   f) In C17, if the relation C9=0 is true, then a 0 will be placed in C17. Otherwise, the value 100 minus the value of C13 will be placed in C13.

Dice Toss (p. 56)

1. Results will vary, but if the dice are tossed a *large* number of times or if the average of the results of the entire class is considered, the results should approximate the following:

   Since a 2 can be obtained in only one way, a 2 should occur 2.78 percent of the time.
   Since a 3 can be obtained in two ways, a 3 should occur 5.56 percent of the time.
   Since a 4 can be obtained in three ways, a 4 should occur 8.33 percent of the time.
   Since a 5 can be obtained in four ways, a 5 should occur 11.11 percent of the time.
   Since a 6 can be obtained in five ways, a 6 should occur 13.89 percent of the time.
   Since a 7 can be obtained in six ways, a 7 should occur 16.67 percent of the time.
   Since an 8 can be obtained in five ways, an 8 should occur 13.89 percent of the time.
   Since a 9 can be obtained in four ways, a 9 should occur 11.11 percent of the time.
   Since a 10 can be obtained in three ways, a 10 should occur 8.33 percent of the time.
   Since an 11 can be obtained in two ways, an 11 should occur 5.56 percent of the time.
   Since a 12 can be obtained in only one way, a 12 should occur 2.78 percent of the time.

2. See answer 1.
3. See answer 1.
4. See answer 1.
5. a) In A7 and C7 random numbers between 1 and 6 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 6*(RAND()) generates a number between 0 and just less than 6, such as 5.9999999. INT(2*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, 3, 4, 5, or 6.

   b) In A11, if the relation C3=0 is true, then a 0 will be placed in A11. Otherwise, the value in A11 will be increased by 1.

c) In A12 through A21, except for resetting to 0 if C3=0, if the sum of the dice is 2 through 12, respectively, the value in the cell is increased by 1. Otherwise it remains as it was.

d) In C11 through C21, except for resetting to 0 if C3=0, determine the percentage by finding the product of 100 and the quotient of the number of tosses of that sum by the number of tosses altogether.

6. Students should reach the conclusion indicated in answer 1.

Birth Month Match (p. 58)

1. Results will vary but there will almost always be at least one pair of people out of ten with the same birth month. The probability of at least one pair of people out of ten with the same birth month is the complement of the probability of no pair in a group of ten people having the same birth month, or almost 100 percent.

1 – (12/12)(11/12)(10/12)(9/12)(8/12)(7/12)(6/12)(5/12)(4/12)(3/12)

= 1 – 0.00386 = 0.996132 = approximately 99.62 percent.

2. See answer 1.

3. See answer 1.

4. a) In E7, if the relation C3=0 is true, a 0 is placed in E7. Otherwise, the value is increased by 1.

b) In B9 through B18, if the relation C3=0 is true, a 0 is placed in the cell. Otherwise, random numbers between 1 and 12 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 12*(RAND()) generates a number between 0 and just less than 12, such as 11.9999999. INT(12*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12.

c) In C9 through C18, if the relation C3=0 is true, a 0 is placed in the cell. If the value (birth month) in the cell to its left matches the value of any other cells in the Birth Month column, a 1 is placed in the cell. Otherwise, a 0 is placed there.

d) In E9, if the relation C3=0 is true, a 0 is placed in the cell. If the sum of the numbers in the Matches column is greater than 0, the value is increased by 1. Otherwise, the value remains as it was.

e) In E11, if the relation C3=0 is true, a 0 is placed in the cell. Otherwise, the product of 100 and the quotient of E9 (Match #) by E7 (Trail #) is indicated in the cell.

5. See answer 1.

Birthday Match (p. 60)

1. Results will vary but the percentage of matches for 100 people should be about 12 percent. The probability of at least one pair of people out of ten with the same birthday is the complement of the probability of no pair in a group of ten people having the same birthday, or almost 12 percent.

1 – (365/365)(364/365)(363/365)(362/365)(361/365)(360/365)(359/365)(358/365)

(357/365)(356/365) = 1–.883051823 = .116948177 = approximately 11.69 percent

2. See answer 1.

3. See answer 1.

4. a) In E7, if the relation C3=0 is true, a 0 is placed in E7. Otherwise, the value is increased by 1.

b) In B9 through B18, if the relation C3=0 is true, a 0 is placed in the cell. Otherwise, random numbers between 1 and 366 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 365*(RAND()) generates a number between 0 and just less

than 365, such as 364.9999999. INT(365*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, ... , or 365.

c) In C9 through C18, if the relation C3=0 is true, a 0 is placed in the cell. If the value (birthday) in the cell to its left matches the value of any other cells in the Birthday column, a 1 is placed in the cell. Otherwise, a 0 is placed there.

d) In E9, if the relation C3=0 is true, a 0 is placed in the cell. If the sum of the numbers in the Matches column is greater than 0, the value is increased by 1. Otherwise, the value remains as it was.

e) In E11, if the relation C3=0 is true, a 0 is placed in the cell. Otherwise, the product of 100 and the quotient of E9 (Match #) by E7 (Trail #) is indicated in the cell.

5. See answer 1.

Card Draw with Replacement (p. 62)

1. Results will vary, but when drawing one card from a deck of fifty-two cards you have 1 chance in 52 of selecting any one card (1.923%).
2. The probability of drawing any suit is 13 out of 52. So, with a large number of draws, say 1000, the file should indicate that about 25 percent of the cards draw were spades. Since there are four cards of each number value, including jack, queen, king, and ace, in a standard deck of fifty-two cards the probability of drawing any one value is 4 out of 52. So about 7.69 percent of the cards drawn should be of any one value.
3. See answers above.
4. a) In B7, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise random numbers between 1 and 4 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 4*(RAND()) generates a number between 0 and just less than 4, such as 3.9999999. INT(4*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, 3, or 4.

b) In G7, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, random numbers between 1 and 13 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 13*(RAND()) generates a number between 0 and just less than 13, such as 12.9999999. INT(13*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, ... , or 13.

c) In B11 through N11, if the relation E3=0 is true, a 0 is placed in the cell. If the random numbers in B7 and G7 exactly match the conditions stated, the cell is increased in value by 1. Otherwise, the cell remains as it was.

d) In O11 through O14, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the sum of the thirteen numbers in the cells to the left of that cell is placed in that cell.

e) In O20 through O23, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the product of 100 and the quotient of the corresponding row sum by the total number of cell draws is calculated to determine the percentage of total draws represented in that row.

f) In B16 through O16, the sum of the four values immediately above it is placed in the cell.

g) In B18 through O18, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the product of 100 and the quotient of the corresponding column sum by the total number of cell draws is calculated to determine the percentage of total draws represented in that column.

Card Draw without Replacement (p. 64)

1. Results will vary, but when drawing one card from a deck of fifty-two cards you have 1 chance in 52 of selecting any one card (1.923%).
2. ...

3. $P(n,r) = n!/(n-r)! = 52!/(52-5)! = 52 \times 51 \times 50 \times 49 \times 48 = 311\,875\,200$

4. $C(n,r) = n!/((n-r)!r!) = 52!/((52-5)!5!) = 52!/(47! \times 5!) = (52 \times 51 \times 50 \times 49 \times 48)/5! = 2\,598\,960$

5. a) In B7, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, random numbers between 1 and 4 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 4*(RAND()) generates a number between 0 and just less than 4, such as 3.9999999. INT(4*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, 3, or 4.

   b) In B7, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, random numbers between 1 and 13 are generated and placed in these cells. The formula RAND() generates a number between 0 and .999999999. 13*(RAND()) generates a number between 0 and just less than 13, such as 12.9999999. INT(13*(RAND() + 1) generates the whole-number portion of one more than the number, a 1, 2, ... , or 13.

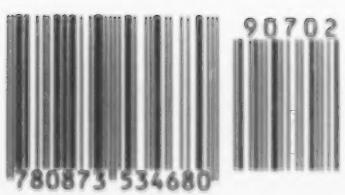
   c) In B11 through N11, if the relation E3=0 is true, a 0 is placed in the cell. If the random numbers in B7 and G7 exactly match the conditions stated, the cell takes a value of 1. Otherwise, the cell remains as it was (which may have been 1 or 0).

   d) In O11 through O14, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the sum of the thirteen numbers in the cells to the left of that cell is placed in that cell.

   e) In O20 through O23, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the product of 100 and the quotient of the corresponding row sum by the total number of cell draws is calculated to determine the percentage of total draws represented in that row.

   f) In B16 through O16, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the sum of the four values immediately above it is placed in the cell.

   g) In B18 through O18, if the relation E3=0 is true, a 0 is placed in the cell. Otherwise, the product of 100 and the quotient of the corresponding column sum by the total number of cell draws is calculated to determine the percentage of total draws represented in that column.



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